CHARACTERISTICS OF SOME ACTIVE NETWORKS

ROBERT E. McCAMEY

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Captain, United States Marine Corps

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ELECTRICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

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ABSTRACT

Some active networks will generate complex poles or zeros in the s-plane. These networks are investigated by root locus techniques in this thesis to ascertain which will yield complex poles and zeros. Those from which complex poles or zeros may be obtained are further investigated to determine characteristics of their root loci.

This thesis is written with the end in mind that these active networks may be used as compensators to provide compensation solutions not easily attainable with passive compensators alone. Analog computer simulation of some of the networks was performed to verify results. Some preliminary investigations are made in solving compensation problems with these active networks.

The author wishes to express his appreciation to Professor George J. Thaler of the U. S. Naval Postgraduate School who suggested the topic for this thesis and without whose advice and encouragement this work could not have been completed.

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TABLE OF SYMBOLS

Symbol	Definition
A_1	Gain of Amplifier 1.
A_2	Gain of Amplifier 2.
α_1, α_2	Attenuation factors of lag networks.
C_v	Output or control voltage.
d	Subscript referring to a zero or pole resulting from a lead network.
F_o	Open loop transfer function.
G_c	Compensator transfer function.
g	Subscript referring to a zero or pole resulting from a lag network.
Γ	The construction angle at point P which locates, on the real axis, the center of the circle of complex poles or zeros producing a constant phase angle at point P.
K	A variable system gain constant in the characteristic equation. Does not include the compensator gain.
K_c	The compensator gain constant.
K_{RL}	The total or root locus gain of the characteristic equation. $K_{RL} = KK_c$.
P	A chosen root of the characteristic equation.
P'	The complex conjugate of P.
ϕ	The construction angle at point P which locates, on the real axis the intersection of the circular arc of complex poles or complex zeros which produce a constant phase angle at point P.
Q	A point on the s-plane at which it is desired to locate a complex compensator pole or zero.
Q'	The complex conjugate of point Q.
R_v	Input or reference voltage.
x	Abscissa of point Q.
y	Ordinate of point Q.

1. Background

Any closed loop system which can be represented by a block diagram has a characteristic equation which may always be manipulated into the form

1.1 $$\frac{K \prod_{i=1}^{n} (s + z_i)}{s^m \prod_{j=1}^{n} (s + p_j)} G_c = -1$$

Where, $0 < K < \infty$, $-\infty < m < \infty$, and the z_i and p_j may be real and/or complex. K is assumed to represent a variable gain element of the system. G_c represents the transfer function of a compensator which may be in any single feed forward or feedback path, or may be cascaded in the main transmission channel. G_c is of the form

1.2 $$G_c = \frac{K_c \prod_{i=1}^{n} (s + z_i)}{s^m \prod_{j=1}^{n} (s + p_j)}$$

The definitions of the terms in Equation 1.2 are the same as Equation 1.1. The total gain of Equation 1.1 is then

1.3 $$K_{RL} = K K_c$$

The selection of a transfer function for G_c, besides being based upon physical realizability is guided by the desire to force the roots of the characteristic equation to lie in such locations on the s-plane that the transient response of the system to be compensated will meet specifications without affecting steady state accuracy. Steady state accuracy specifications are normally met before compensation by setting system gain to obtain an acceptable steady state error coefficient. However, it is conceivable that compensation could be required to meet steady state specifications.

It is common practice to select two complex conjugate root locations and then to choose G_c so that the two selected locations will be roots of the characteristic equation. Generally, it is hoped that the selected

roots will prove to be dominant, but a certain amount of experience and foresight is required in order to ensure that this end will be attained. Even with experience, some trial and error is frequently involved. In any case, only calculation of the transient response will determine whether or not a particular solution is acceptable. The concept of dominance [1] states that if the residues at certain roots of the characteristic equation are large compared to the residues at all other roots, and if the real parts of those certain roots are several times smaller than the real parts of any other roots, then those certain roots are dominant. A transient response calculated using only the dominant roots will be sufficiently accurate for engineering purposes. All roots must be included however when calculating the phase angle of the transient response. Note that this concept may significantly reduce the labor involved in calculating the transient response of higher order systems.

The common use of passive networks to effect compensation is with G_c containing only negative real poles and zeros, and usually with $K \lesssim 1$. It is possible to generate complex poles or zeros with passive networks [2,5], but it is difficult to use these networks practically due to large attenuation in the compensator and extreme sensitivity of the complex pole or zero location to variations of components within the network. In some cases an additional amplifier is required to correct for the attenuation of a passive compensator, so the compensator could have been designed as an active compensator from the start.

2. Introduction to Active Networks

The terms "active network" and "active compensator" as used in this thesis are interchangable and refer to networks which contain passive components, a summer, and one or two amplifiers.

Frequently, complex poles and/or complex zeros are present in a system before compensation is attempted. Having networks available to use as compensators which will produce complex poles or zeros will in some instances make the compensation problem easier. The active networks investigated in this thesis are flexible in that the complex poles or zeros resulting from them may be relocated on the s-plane by varying either an amplifier gain or the magnitude of one or more of the passive elements in the network.

The passive networks, which are essential to active compensators, may be either RC or RL networks. However, RL networks pose some problems due to difficulties in determining and allowing for the resistance of the inductive elements.

Active compensation is not envisioned as replacing passive compensation, but rather as supplementing it by providing solutions to compensation problems not easily attainable with passive compensators alone. In some instances it may prove desirable to combine active and passive compensators to achieve desired results.

Two major disadvantages of active compensators are cost and complexity. Both of these factors arise from the fact that amplifiers and summers are required as parts of the compensator.

The passive networks, their transfer functions, and s-plane pole-zero arrays which will be used as basic components in the active networks are depicted in Fig. 2-1. They are held to simple configurations primarily to provide attainable goals for this investigation. In addition,

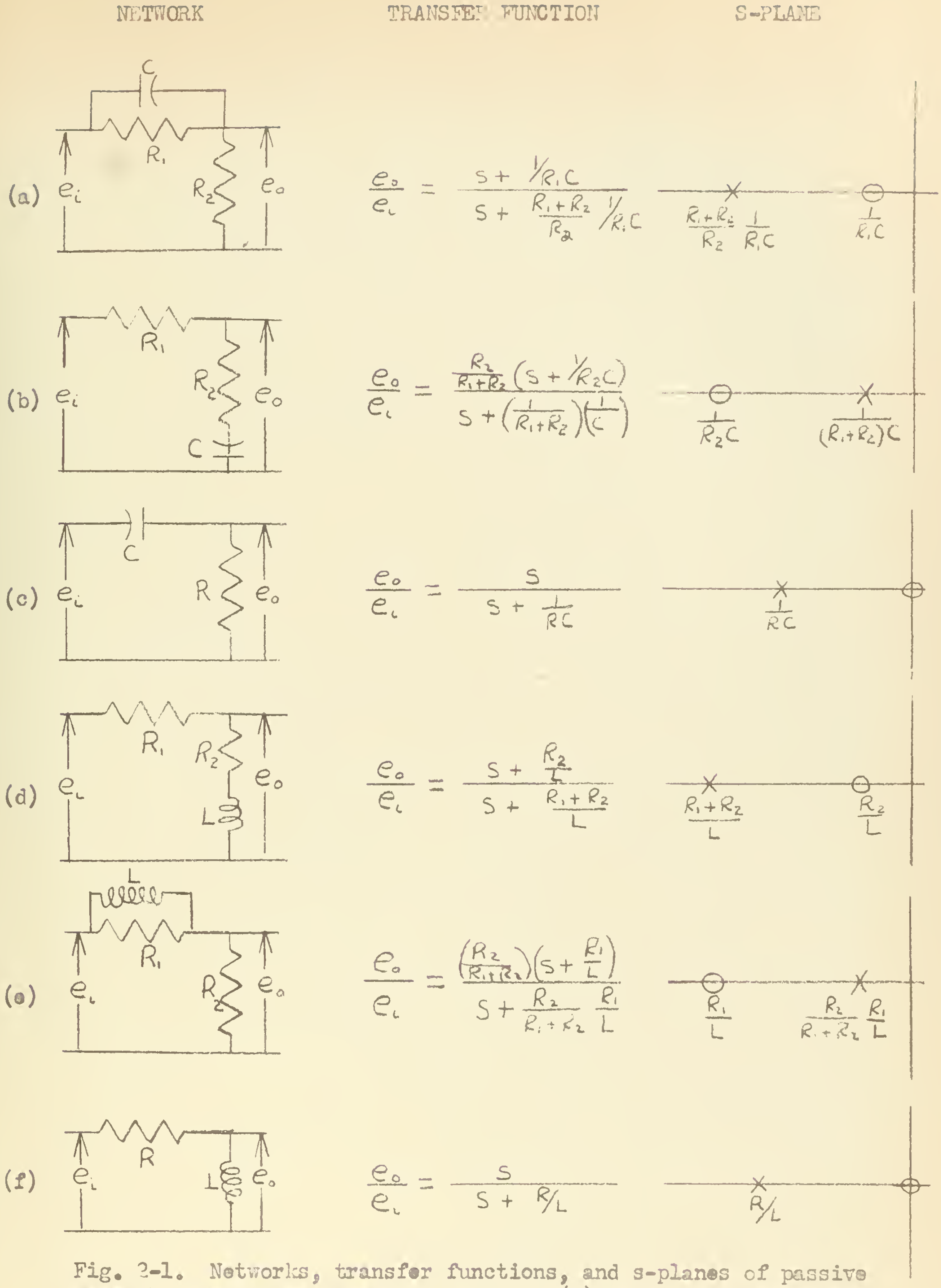

Fig. 2-1. Networks, transfer functions, and s-planes of passive networks used. The network of Fig. 2-1(c) cannot ordinarily be used in the main transmission path unless there exists a parallel forward path through which dc components may pass.

all compensation problems attempted could be solved using compensators which required only these simple networks. The transfer functions in Fig. 2-1 are defined on Table 2-1 for algebraic manipulation.

TABLE 2-1.

$$\frac{s + \frac{1}{R_1 C}}{s + \frac{R_1 + R_2}{R_2} \frac{1}{R_1 C}} \triangleq \frac{s + z_d}{s + p_d}$$

$$\frac{\frac{R_2}{R_1 + R_2}\left(s + \frac{1}{R_2 C}\right)}{s + \frac{1}{(R_1 + R_2)C}} \triangleq \frac{\alpha\left(s + z_g\right)}{(s + p_g)}$$

$$\frac{s}{1 + \frac{1}{RC}} \triangleq \frac{s}{s + z_d}$$

$$\frac{s + \frac{R_2}{L}}{s + \frac{R_1 + R_2}{L}} \triangleq \frac{s + z_d}{s + p_d}$$

$$\frac{\frac{R_2}{R_1 + R_2}\left(s + \frac{R_1}{L}\right)}{s + \frac{R_2}{R_1 + R_2} \frac{R_1}{L}} \triangleq \frac{\alpha\left(s + z_g\right)}{(s + p_g)}$$

$$\frac{s}{s + R/L} \triangleq \frac{s}{s + p_d}$$

Definitions of the transfer functions of Fig. 2-1 for algebraic manipulation. The subscript d refers to poles or zeros resulting from a lead network. The subscript g refers to poles or zeros resulting from a lag network.

5. Investigation of Some Active Networks to Determine Which Will Yield Complex Poles or Zeros

Not all active networks will yield complex poles or zeros. It is therefore necessary to investigate several combinations of amplifiers, summers, and passive networks to determine some of the configurations which will produce the desired complex poles and zeros.

Two basic network configurations, one a parallel feed forward and the other a feedback, were selected for investigation. Solutions were obtained by placing a lag or a lead transfer function and amplifier in each path available. The overall transfer function of the network was then determined and investigated for the existence of complex poles or zeros.

Fifteen different networks were investigated using this plan. Some were unsuccessful in that they did not yield the desired complex poles and zeros. Notwithstanding, all fifteen are presented in this section so that other investigators attempting to obtain similar results need not expend time and energy in fruitless areas.

There are, undoubtedly other simple active networks which will produce complex poles and zeros. However, it was impossible to investigate all possibilities within the scope of this thesis. Some network configurations which may prove interesting in future investigations of this nature are presented in Appendix B.

3(A). Lead-lead feed forward summing compensator.

Fig. 3-1 is the block diagram of this compensator. The transfer function of Fig. 3-1 is

$$3.1 \qquad \frac{V_o}{V_i} = \frac{A_1(S+Z_{d_1})(S+P_{d_2}) + A_2(S+Z_{d_2})(S+P_{d_1})}{(S+P_{d_1})(S+P_{d_2})}$$

For factoring by root locus, the numerator of Equation 3.1 may be manipulated into the form

$$3.2 \qquad \frac{A_1(S+Z_{d_1})(S+P_{d_2})}{A_2(S+Z_{d_2})(S+P_{d_1})} = -1$$

The root locus of Equation 3.2 is Fig. 3-2. By inspection of Fig. 3-2 it can be seen that there is no possible way to obtain complex roots from Equation 3.2. Therefore this compensator is of no further interest in this thesis.

3(B). Lead-lead feed forward difference compensator.

Fig. 3-3 is the block diagram of this compensator. The transfer function of Fig. 3-3 is

$$3.3 \qquad \frac{V_o}{V_i} = \frac{A_1(S+Z_{d_1})(S+P_{d_2}) - A_2(S+Z_{d_2})(S+P_{d_1})}{(S+P_{d_1})(S+P_{d_2})}$$

For factoring by root locus, the numerator of Equation 3.3 may be manipulated into the form

$$3.4 \qquad \frac{A_1(S+Z_{d_1})(S+P_{d_2})}{A_2(S+Z_{d_2})(S+P_{d_1})} = 1$$

Note that Equation 3.4 is a zero degree root locus. The possible root loci arising from Equation 3.4 are Figs. 3-4, 3-5, 3-6, and 3-7. It can be seen from Figs. 3-4 and 3-5 that complex roots of Equation 3.4 will obtain for certain values of $\frac{A_1}{A_2}$. These complex roots will become complex zeros in Equation 3.3 and result in Equation 3.3 having the form

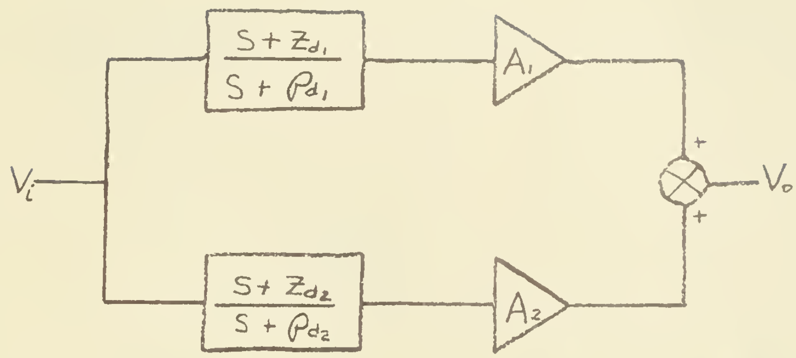

Fig. 3-1. Block diagram of the lead-lead feed forward summing compensator.

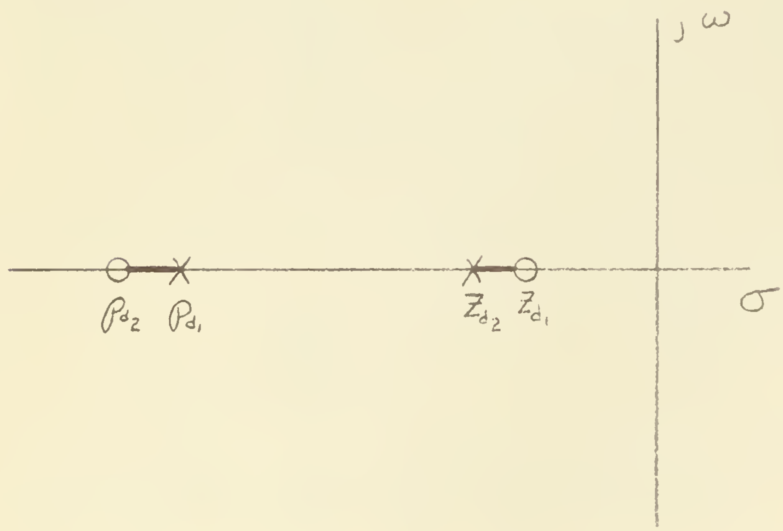

Fig. 3-2. Root locus of Equation 3.2.

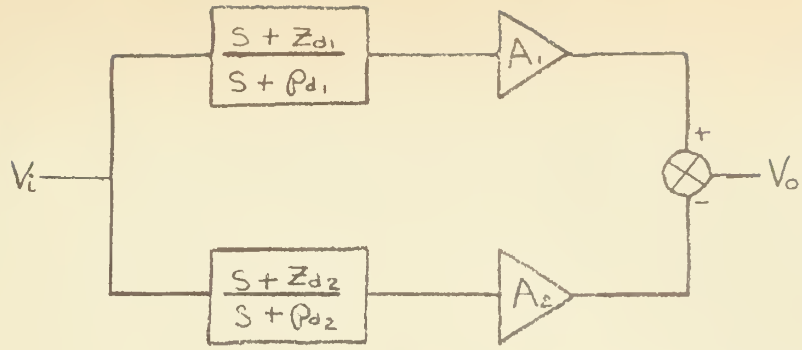

Fig. 3-3. Block diagram of the lead-lead feed forward difference compensator.

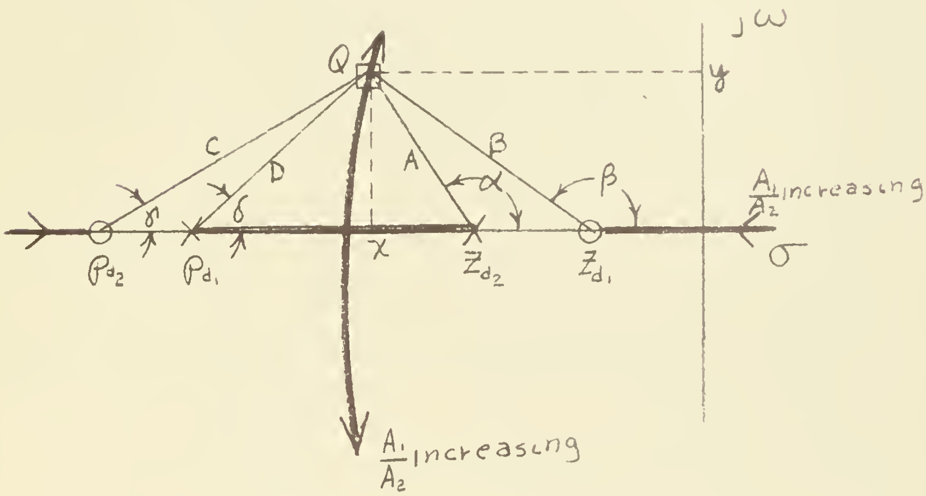

Fig. 3-4. A possible root locus of Equation 3.4.

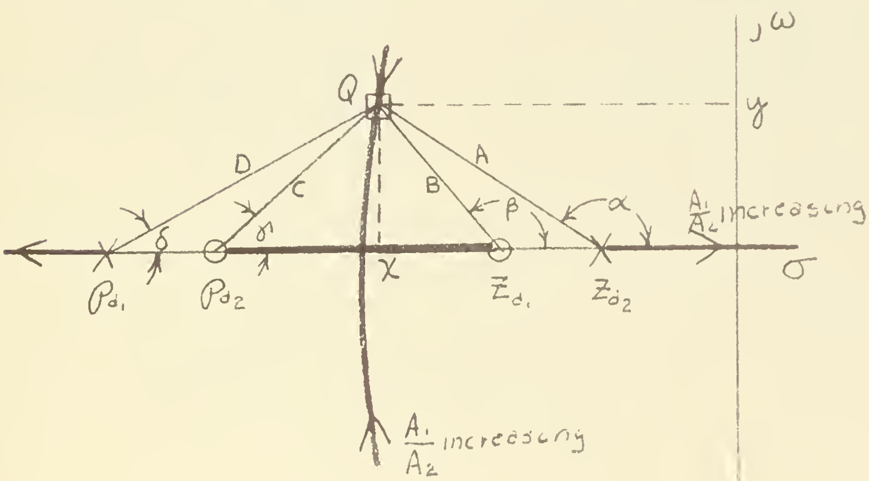

Fig. 3-5. A possible root locus of Equation 3.4.

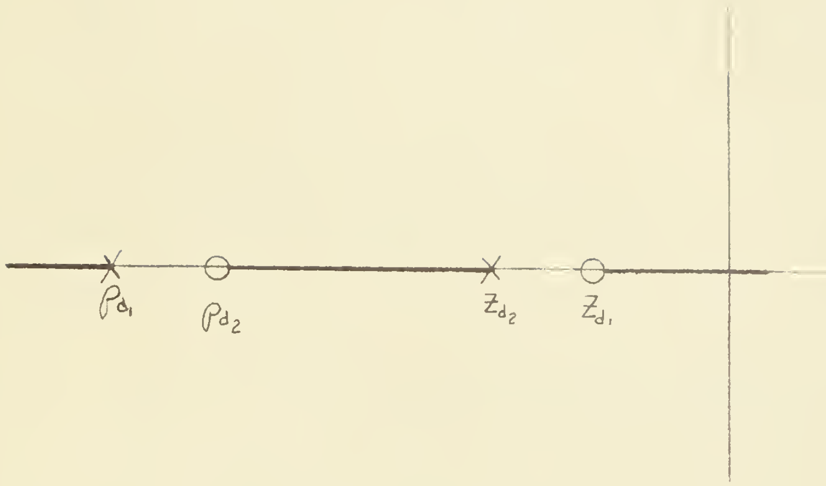

Fig. 3-6. A possible root locus of Equation 3.4.

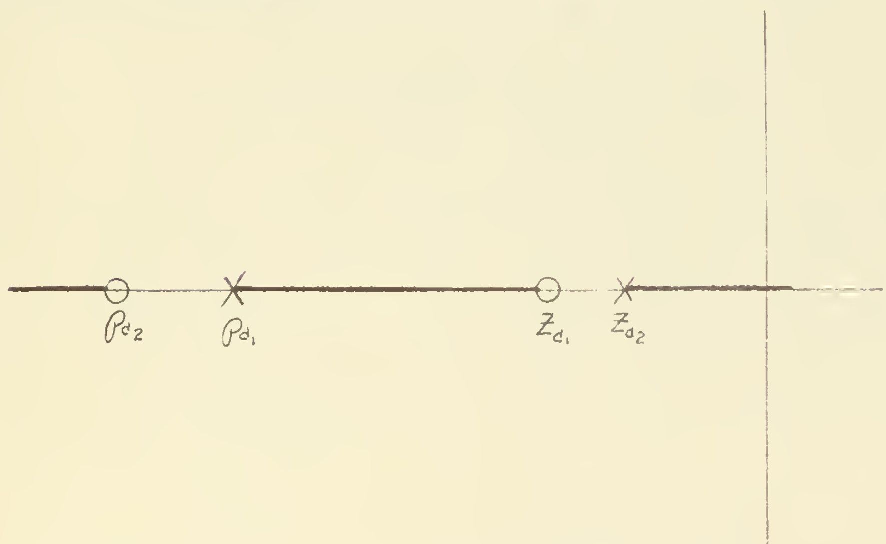

Fig. 3-7. A possible root locus of Equation 3.4.

3.5 $$\frac{V_o}{V_i} = \frac{(A-A_2)(s+\sigma+j\omega)(s+\sigma-j\omega)}{(s+p_{d_1})(s+p_{d_2})}$$

This transfer function is of further interest because p_{d_1} and p_{d_2}, the real poles, may be placed far out in the left half plane if desired.

There can be no complex roots resulting from the root loci of Figs. 3-6 or 3-7. Therefore they are of no further interest in this investigation.

3(C). Lead-lead negative feedback compensator.

Fig. 3-8 is the block diagram of this compensator. The transfer function of Fig. 3-8 is

3.6 $$\frac{V_o}{V_i} = \frac{(s+z_{d_1})(s+p_{d_2})}{(s+p_{d_1})(s+p_{d_2})+A_1(s+z_{d_1})(s+z_{d_2})}$$

For factoring by root locus, the denominator of Equation 3.6 may be manipulated into the form

3.7 $$\frac{A_1(s+z_{d_1})(s+z_{d_2})}{(s+p_{d_1})(s+p_{d_2})} = -1$$

Fig. 3-9 is the root locus of Equation 3.7. Any locations of p_{d_1}, p_{d_2}, z_{d_1}, or z_{d_2} will produce the locus of Fig. 3-9 unless $|z_{d_2}| > |p_{d_1}|$ or $|z_{d_1}| > |p_{d_2}|$. The complex roots which obtain for certain values of A_1 will become complex poles in Equation 3.6 and result in that equation having the form

3.8 $$\frac{V_o}{V_i} = \frac{(s+z_{d_1})(s+p_{d_2})}{(A_1+1)(s+\sigma+j\omega)(s+\sigma-j\omega)}$$

This transfer function is of further interest because one of the real zeros of Equation 3.8 may be placed close to the origin while the other may be placed as far out in the left half plane as desired. Note from Fig. 3-9 that the complex poles will always be in the left half s-plane.

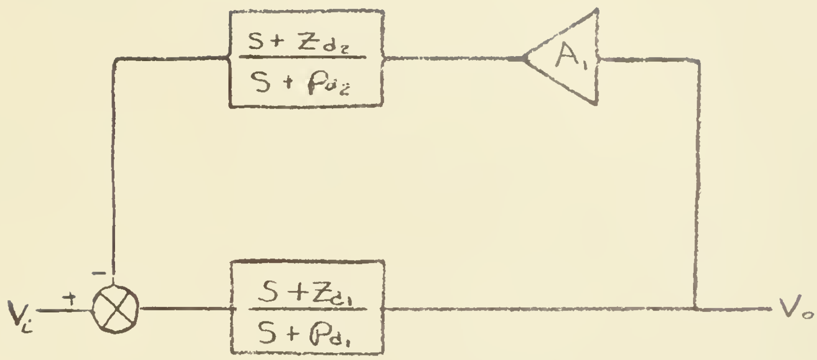

Fig. 3-8. Block diagram of the lead-lead negative feedback compensator.

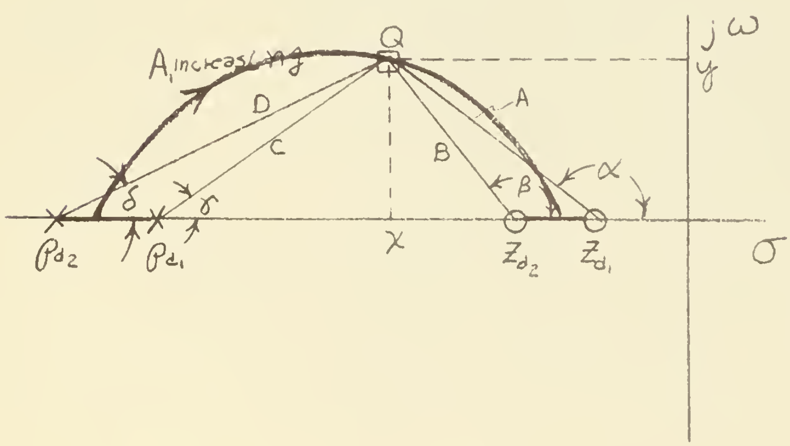

Fig. 3-9. A possible root locus of Equation 3.7.

3(D). Lead-lead positive feedback compensator.

Fig. 3-10 is the block diagram of this compensator. The transfer function of Fig. 3-10 is

3.9 $$\frac{V_o}{V_i} = \frac{(s+z_{d_1})(s+p_{d_2})}{(s+p_{d_1})(s+p_{d_2}) - A_1(s+z_{d_1})(s+z_{d_2})}$$

For factoring by root locus, the denominator of Equation 3.9 may be manipulated into the form

3.10 $$\frac{A_1(s+z_{d_1})(s+z_{d_2})}{(s+p_{d_1})(s+p_{d_2})} = 1$$

Note that Equation 3.10 is a zero degree root locus. Fig. 3-11 is the root locus of Equation 3.10. It can be seen by inspection of Fig. 3-11 that there is no way to obtain complex roots from Equation 3.10. Therefore this compensator is of no further interest in this investigation

3(E). Lag-lag feed forward summing compensator.

Fig. 3-12 is the block diagram of this compensator. The transfer function of Fig. 3-12 is

3.11 $$\frac{V_o}{V_i} = \frac{A_1\alpha_1(s+z_{g_1})(s+p_{g_2}) + A_2\alpha_2(s+z_{g_2})(s+p_{g_1})}{(s+p_{g_1})(s+p_{g_2})}$$

For factoring by root locus, the numerator of Equation 3.11 may be manipulated into the form

3.12 $$\frac{A_1\alpha_1(s+z_{g_1})(s+p_{g_2})}{A_2\alpha_2(s+z_{g_2})(s+p_{g_1})} = -1$$

Fig. 3-13 is the root locus of Equation 3.12. By inspection of Fig. 3-13 it can be seen that there is no possible way to obtain complex roots from Equation 3.12. Therefore this compensator is of no further interest in this investigation.

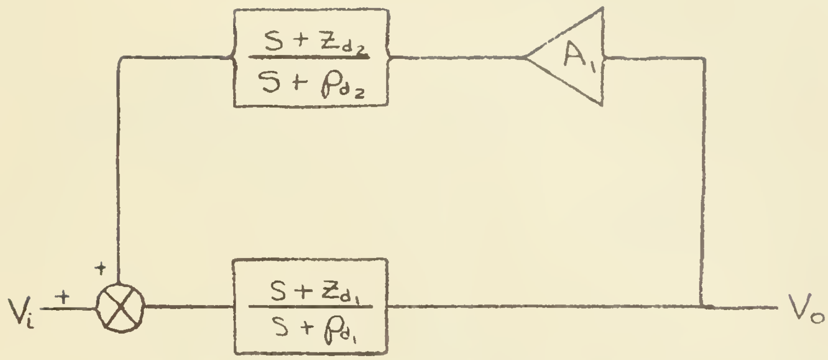

Fig. 3-10. Block diagram of the lead-lead positive feedback compensator.

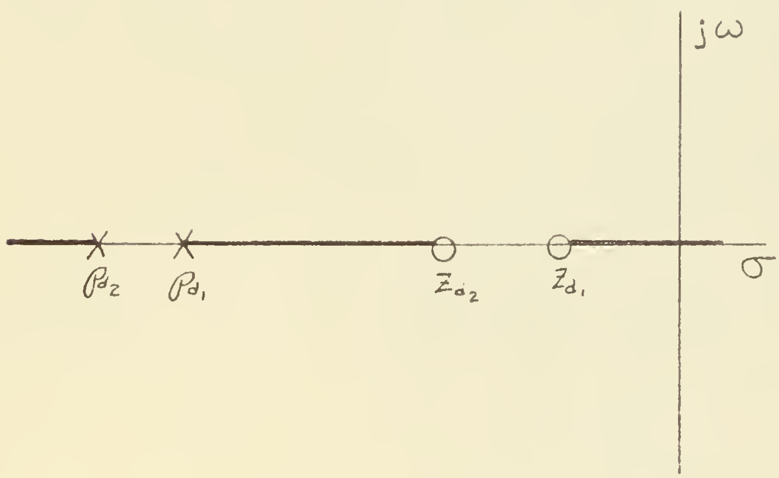

Fig. 3-11. A possible root locus of Equation 3.10.

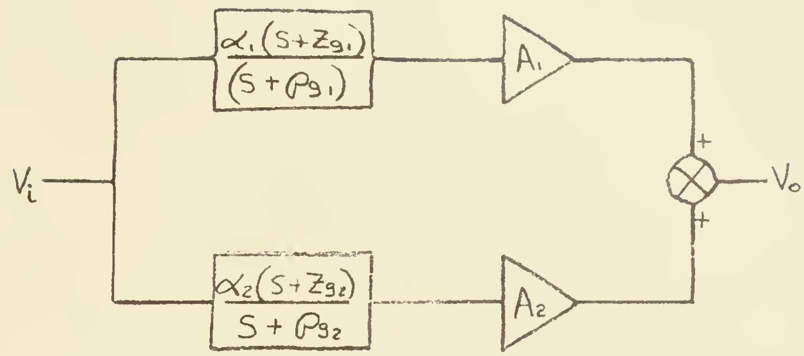

Fig. 3-12. Block diagram of lag-lag feed forward summing compensator.

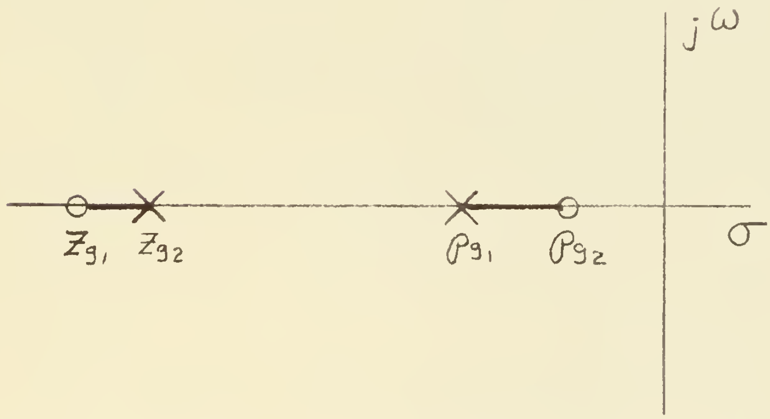

Fig. 3-13. A possible root locus of Equation 3.12.

3(F). Lag-lag feed forward difference compensator.

Fig. 3-14 is the block diagram of this compensator. The transfer function of Fig. 3-14 is

$$3.13 \qquad \frac{V_o}{V_i} = \frac{A_1\alpha_1(s+z_{g_1})(s+p_{g_2}) - A_2\alpha_2(s+z_{g_2})(s+p_{g_1})}{(s+p_{g_1})(s+p_{g_2})}$$

For factoring by root locus, the numerator of Equation 3.13 may be manipulated into the form

$$3.14 \qquad \frac{A_1\alpha_1(s+z_{g_1})(s+p_{g_2})}{A_2\alpha_2(s+z_{g_2})(s+p_{g_1})} = 1$$

Note that Equation 3.14 is a zero degree root locus. The possible root loci arising from Equation 3.14 are Figs. 3-15, 3-16, 3-17, and 3-18. It can be seen from Figs. 3-15 and 3-16 that complex roots of Equation 3.14 will obtain for certain values of $\frac{A_1\alpha_1}{A_2\alpha_2}$. These complex roots will become complex zeros in Equation 3.13 and will result in the equation having the form

$$3.15 \qquad \frac{V_o}{V_i} = \frac{(A_1\alpha_1 - A_2\alpha_2)(s+\sigma+j\omega)(s+\sigma-j\omega)}{(s+p_{g_1})(s+p_{g_2})}$$

This transfer function is of further interest because p_{g_1}, and p_{g_2}, the real poles of Equation 3.15, may be placed fairly close to the origin in the left hand s-plane.

There can be no complex roots resulting from the root loci of Figs. 3-17 and 3-18. Therefore they are of no further interest in this investigation.

3(G). Lag-lag negative feedback compensator.

Fig. 3-19 is the block diagram of this compensator. The transfer function of Fig. 3-19 is

$$3.16 \qquad \frac{V_o}{V_i} = \frac{\alpha_1(s+z_{g_1})(s+p_{g_2})}{(s+p_{g_1})(s+p_{g_2}) + A_1\alpha_1\alpha_2(s+z_{g_1})(s+z_{g_2})}$$

For factoring by root locus, the denominator of Equation 3.16 may be manipulated into the form

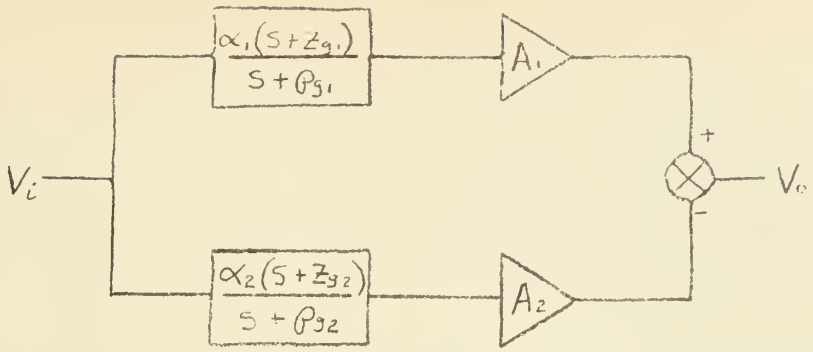

Fig. 3-14. Block diagram of the lag-lag feed forward difference compensator.

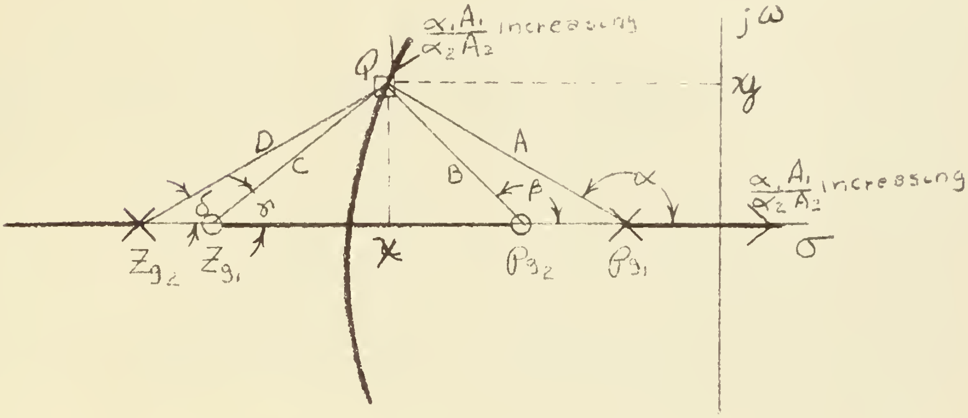

Fig. 3-15. A possible root locus of Equation 3.14.

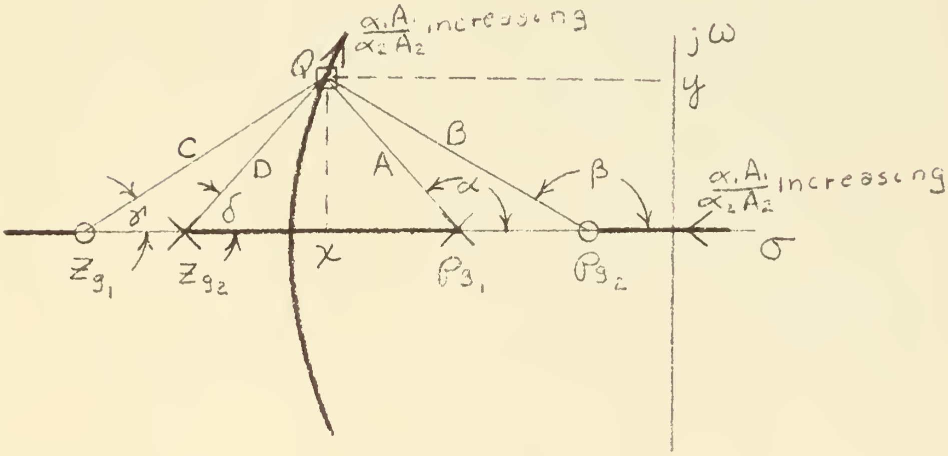

Fig. 3-16. A possible root locus of Equation 3.14.

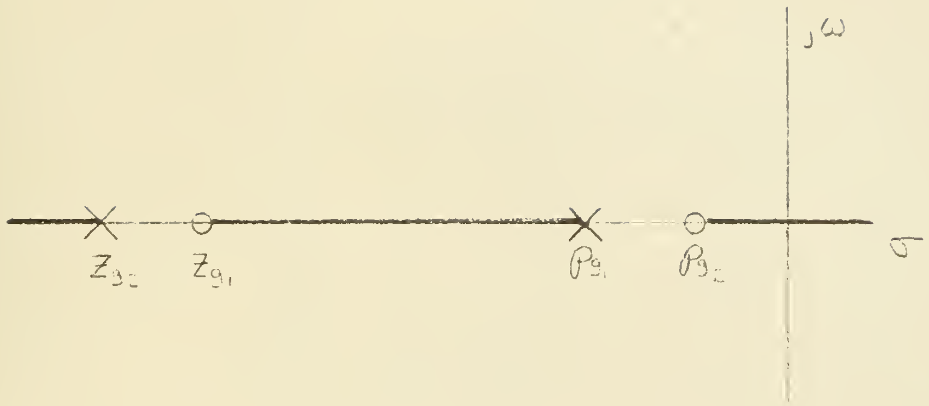

Fig. 3-17. A possible root locus of Equation 3.14.

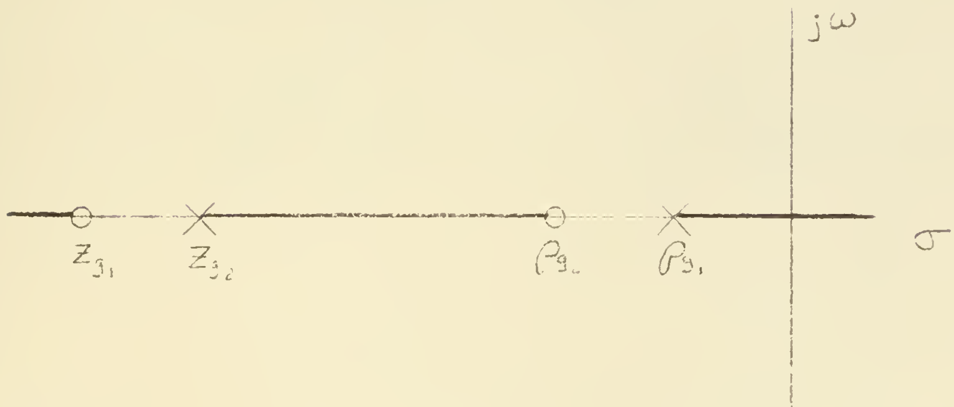

Fig. 3-18. A possible root locus of Equation 3.14.

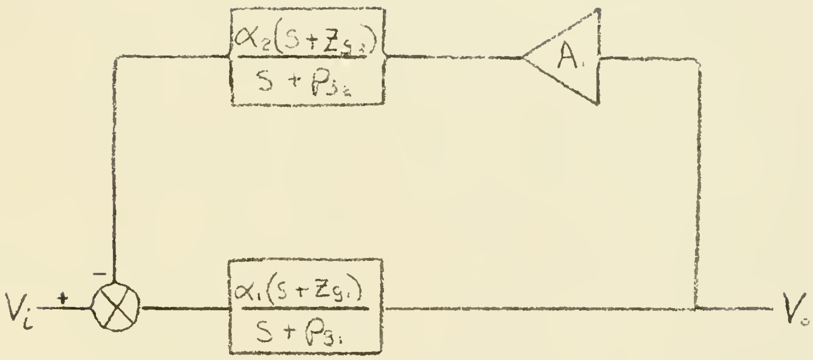

Fig. 3-19. Block diagram of the lag-lag negative feedback compensator.

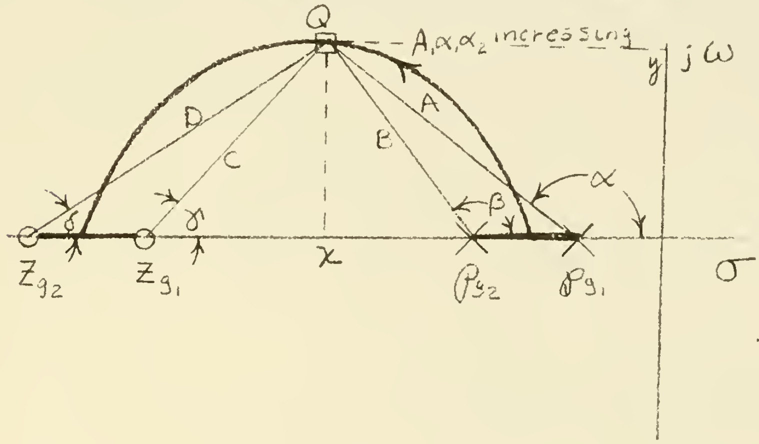

Fig. 3-20. A possible root locus of Equation 3.17.

3.17 $$\frac{A_1\alpha_1\alpha_2(s+z_{g_1})(s+z_{g_2})}{(s+p_{g_1})(s+p_{g_2})} = -1$$

The root locus of Equation 3.17 is Fig. 3-20. It can be seen from Fig. 3-20 that complex roots of Equation 3.17 will obtain for certain values of $A_1\alpha_1\alpha_2$. These complex roots will become complex poles in Equation 3.16 and will result in that equation having the form

3.18 $$\frac{V_o}{V_i} = \frac{\alpha_1(s+z_{g_1})(s+p_{g_2})}{(A_1\alpha_1\alpha_2+1)(s+\sigma+j\omega)(s+\sigma-j\omega)}$$

This transfer function is of further interest because one of the real zeros of Equation 3.18 may be placed near the origin while the other may be as far out in the left hand s-plane as desired. The complex poles will always be in the left hand s-plane.

Note that Fig. 3-20 will be the root locus of Equation 3.17 unless $|p_{g_2}| > |z_{g_1}|$ or $|p_{g_1}| > |z_{g_2}|$. In neither of these cases could complex roots be obtained from Equation 3.17. They are therefore of no interest in this investigation.

3(H). Lag-lag positive feedback compensator.

Fig. 3-21 is the block diagram of this compensator. The transfer function of Fig. 3-21 is

3.19 $$\frac{V_o}{V_i} = \frac{\alpha_1(s+z_{g_1})(s+p_{g_2})}{(s+p_{g_1})(s+p_{g_2}) - A_1\alpha_1\alpha_2(s+z_{g_1})(s+z_{g_2})}$$

For factoring by root locus, the denominator of Equation 3.19 may be manipulated into the form

3.20 $$\frac{A_1\alpha_1\alpha_2(s+z_{g_1})(s+z_{g_2})}{(s+p_{g_1})(s+p_{g_2})} = 1$$

Note that Equation 3.20 is a zero degree root locus. The root locus of Equation 3.20 is Fig. 3-22. By inspection of Fig. 3-22 it can be seen

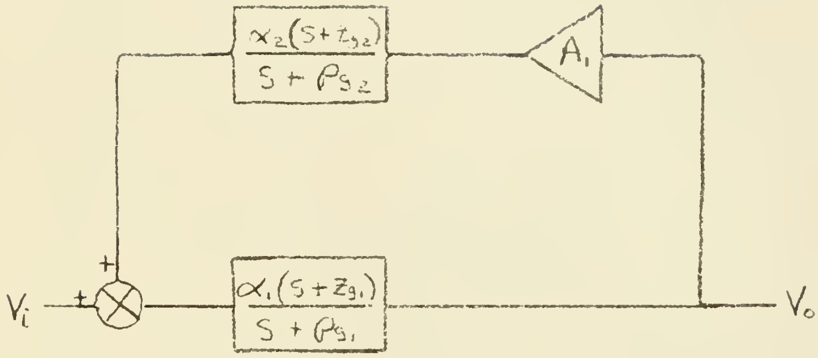

Fig. 3-21. Block diagram of the lag-lag positive feedback compensator.

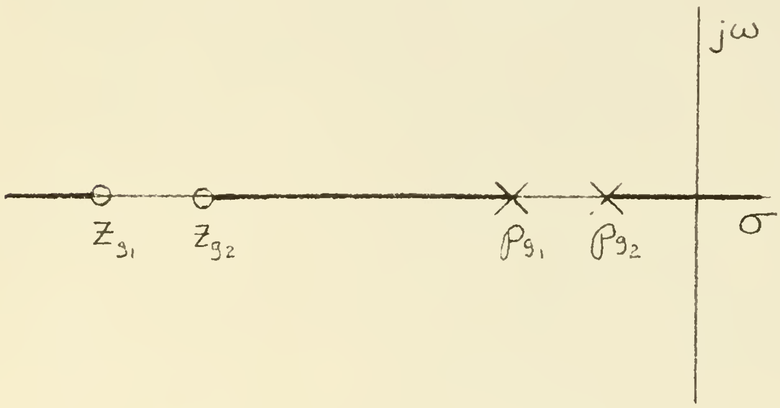

Fig. 3-22. A possible root locus of Equation 3.20.

that there is no possible way to obtain complex roots from Equation 3.20. Therefore this compensator is of no further interest in this investigation.

3(I). Lag-lead feed forward summing compensator.

The block diagram of this compensator is Fig. 3-23. The transfer function of Fig. 3-23 is

3.21 $$\frac{V_o}{V_i} = \frac{A_1\alpha(s+z_g)(s+p_d)+A_2(s+z_d)(s+p_g)}{(s+p_d)(s+p_g)}$$

For factoring by root locus, the numerator of Equation 3.21 may be manipulated into the form

3.22 $$\frac{A_1\alpha(s+z_g)(s+p_d)}{A_2(s+z_d)(s+p_g)} = -1$$

Fig. 3-24 is the root locus of Equation 3.22. It can be seen by inspection of Fig. 3-24 that complex roots will obtain for certain values of $\frac{A_1\alpha}{A_2}$ so long as $|z_g|>|z_d|$ and $|p_d|>|p_g|$. These complex roots will become complex zeros in Equation 3.21 resulting in that equation having the form

3.23 $$\frac{V_o}{V_i} = \frac{(A_1\alpha+A_2)(s+\sigma+j\omega)(s+\sigma-j\omega)}{(s+p_g)(s+p_d)}$$

This transfer function is of further interest because one of the real poles of Equation 3.23 may be placed close to the origin in the left hand s-plane while the other may be placed as far out on the negative real axis as desired. The complex zeros will always be in the left hand s-plane.

3(J). Lag-lead feed forward difference compensator;
lag in the negative path.

The block diagram of this compensator is Fig. 3-25. The transfer function of Fig. 3-25 is

3.24 $$\frac{V_o}{V_i} = \frac{-A_1\alpha(s+z_g)(s+p_d)+A_2(s+z_d)(s+p_g)}{(s+p_d)(s+p_g)}$$

For factoring by root locus, the numerator of Equation 3.24 may be manip-

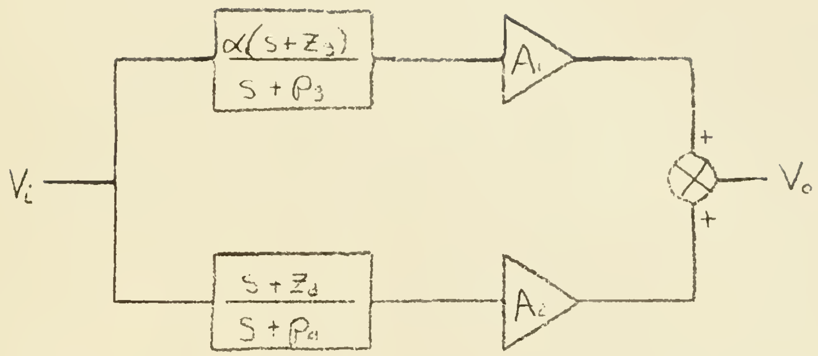

Fig. 3-23. Block diagram of lag-lead feed forward summing compensator.

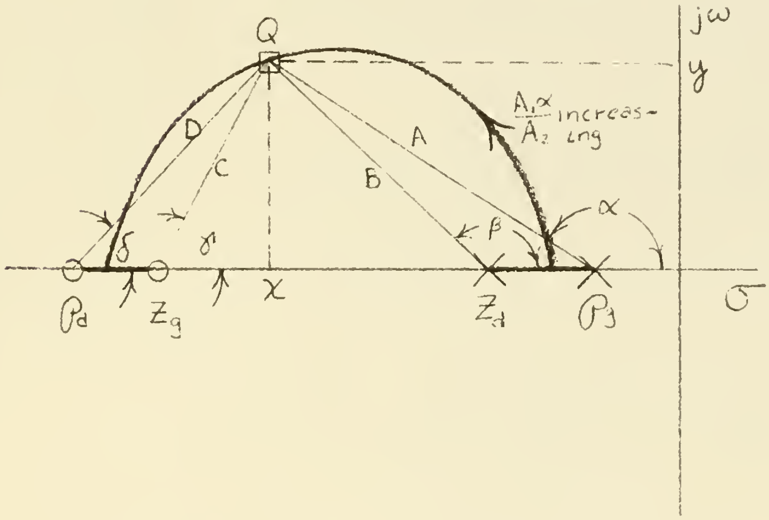

Fig. 3-24. A possible root locus of Equation 3.22.

ulated into the form

$$3.25 \qquad \frac{A_1 \alpha (s + z_g)(s + \rho_d)}{A_2 (s + z_d)(s + \rho_g)} = 1$$

Note that Equation 3.25 is a zero degree root locus. The root locus of Equation 3.25 is Fig. 3-26. By inspection of Fig. 3-26 it can be seen that there is no possible way to obtain complex roots from Equation 3.25. Therefore this compensator is of no further interest in this investigation.

3(K). Lag-lead feed forward difference compensator: lead in the negative path.

It can be seen by inspection of Fig. 3-25 and Equations 3.24 and 3.25 that there is no change resulting from reversing the algebraic signs of the lag and lead paths. Therefore this compensator is of no further interest in this investigation.

3(L). Lag-lead negative feedback compensator; lag in the feedback path.

The block diagram of this compensator is Fig. 3-27. The transfer function of Fig. 3-27 is

$$3.26 \qquad \frac{V_o}{V_i} = \frac{(s + z_d)(s + \rho_g)}{(s + \rho_d)(s + \rho_g) + A_1 \alpha (s + z_g)(s + z_d)}$$

For factoring by root locus, the denominator of Equation 3.26 may be manipulated into the form

$$3.27 \qquad \frac{A_1 \alpha (s + z_d)(s + z_g)}{(s + \rho_d)(s + \rho_g)} = -1$$

The root locus of Equation 3.27 is Fig. 3.28. By inspection of Fig. 3.28 it can be seen that there is no possible way to obtain complex roots of Equation 3.27. Therefore, this compensator is of no further interest in this investigation.

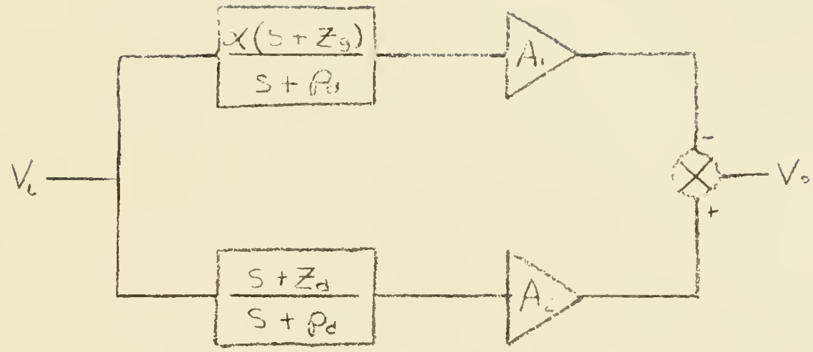

Fig. 3-25. Block diagram of the lag-lead feed forward difference compensator. Lag in the negative path.

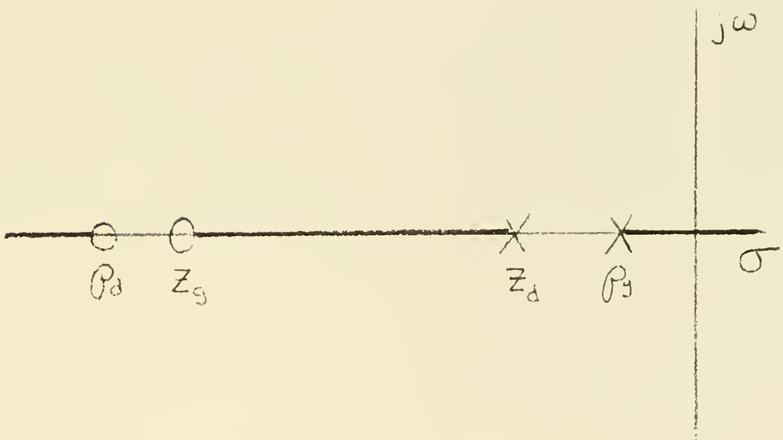

Fig. 3-26. A possible root locus of Equation 3.25.

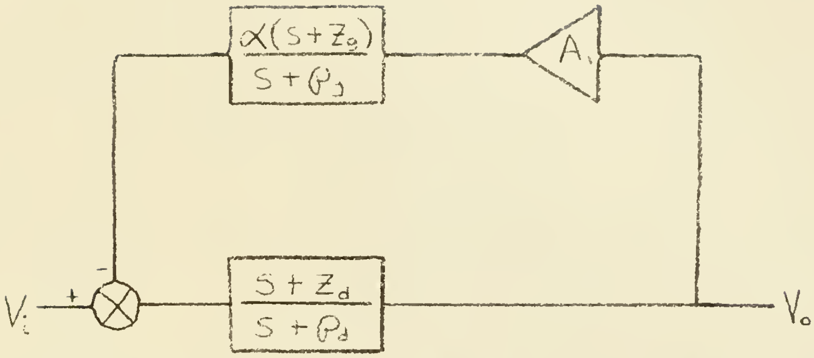

Fig. 3-27. Block diagram of the lag-lead negative feedback compensator. Lag in the feedback path.

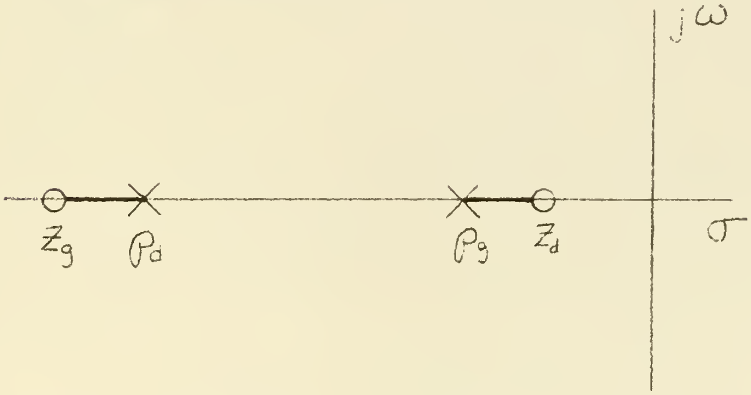

Fig. 3-28. A possible root locus of Equation 3.27.

3(M). Lag-lead negative feedback compensator;
lead in the feedback path.

The block diagram of this compensator is Fig. 3-29. The transfer function of Fig. 3-29 is

3.28 $$\frac{V_o}{V_i} = \frac{(s+z_g)(s+\rho_d)}{(s+\rho_g)(s+\rho_d) + A_1\alpha(s+z_d)(s+z_g)}$$

For factoring by root locus, the denominator of Equation 3.28 can be manipulated into the form

3.29 $$\frac{A_1\alpha(s+z_d)(s+z_g)}{(s+\rho_g)(s+\rho_d)} = -1$$

Equation 3.29 is the same as Equation 3.27 and will yield identical results. Therefore this compensator is of no further interest in this investigation.

3(N). Lag-lead positive feedback compensator;
lag in the feedback path.

The block diagram of this compensator is Fig. 3-30. The transfer function of Fig. 3-30 is

3.30 $$\frac{V_o}{V_i} = \frac{(s+z_d)(s+\rho_g)}{(s+\rho_g)(s+\rho_d) - A_1\alpha(s+z_g)(s+z_d)}$$

For factoring by root locus, the denominator of Equation 3.30 can be manipulated into the form

3.31 $$\frac{A_1\alpha(s+z_g)(s+z_d)}{(s+\rho_g)(s+\rho_d)} = 1$$

Note that Equation 3.31 is a zero degree root locus. Possible root loci resulting from Equation 3.31 are Figs. 3-31, 3-32, 3-33, and 3-34. It can be seen by inspection of Figs. 3-33 and 3-34 that complex roots of Equation 3.31 will obtain for certain values of $A_1\alpha$. These complex roots will become complex poles in Equation 3.30 and result in that equation having the form

3.32 $$\frac{V_o}{V_i} = \frac{(s+z_d)(s+\rho_g)}{(1-A_1\alpha)(s+\sigma+j\omega)(s+\sigma-j\omega)}$$

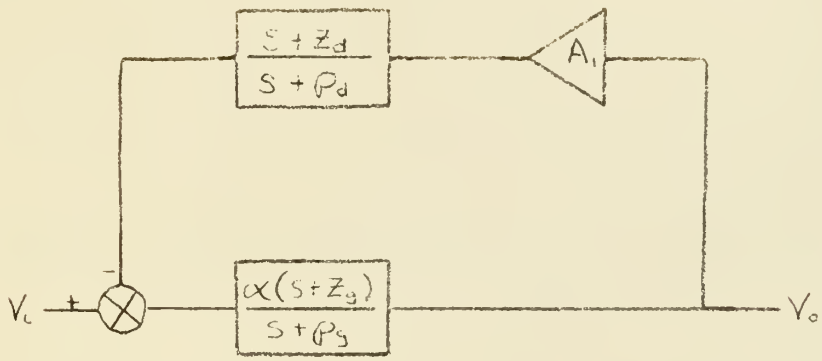

Fig. 3-29. Block diagram of the lag-lead negative feedback compensator. Lead in the feedback path.

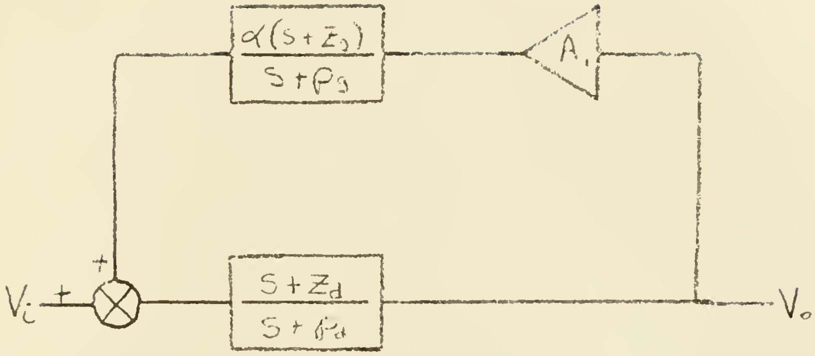

Fig. 3-30. Block diagram of the lag-lead positive feedback compensator. Lag in the feedback path.

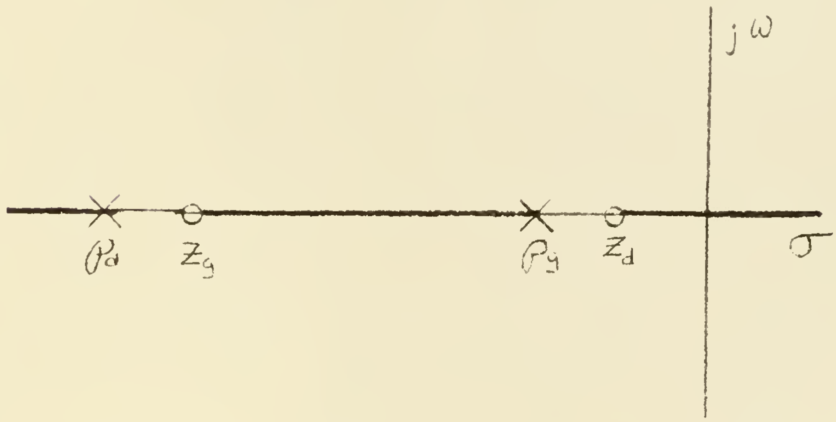

Fig. 3-31. A possible root locus of Equation 3.31.

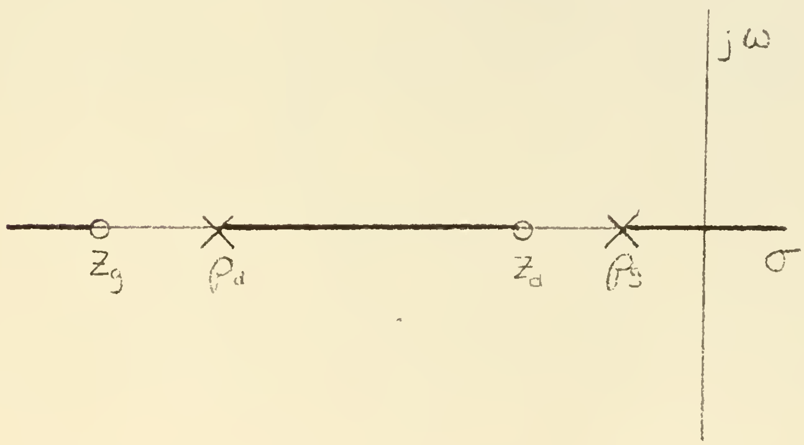

Fig. 3-32. A possible root locus of Equation 3.31.

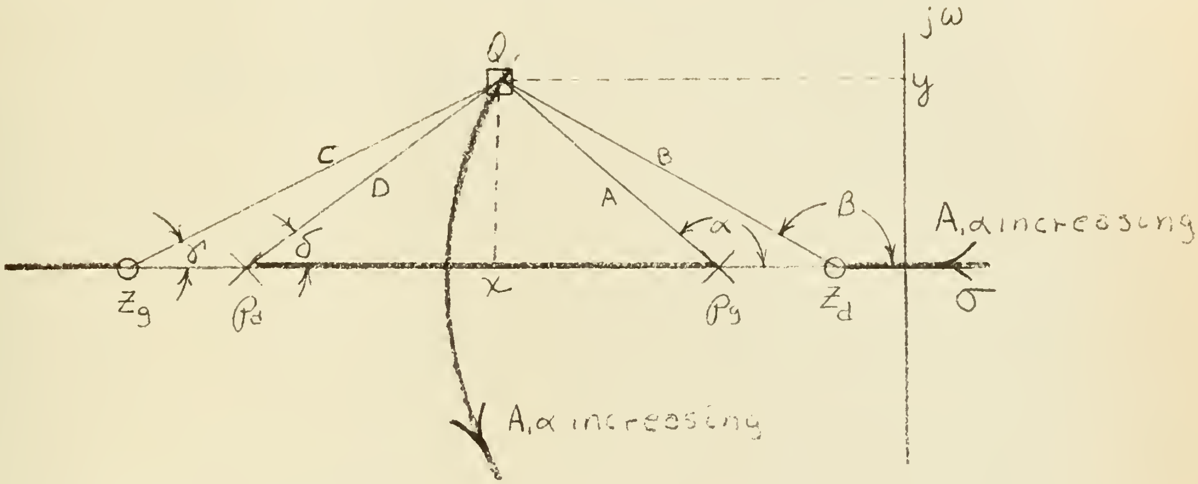

Fig. 3-33. A possible root locus of Equation 3.31.

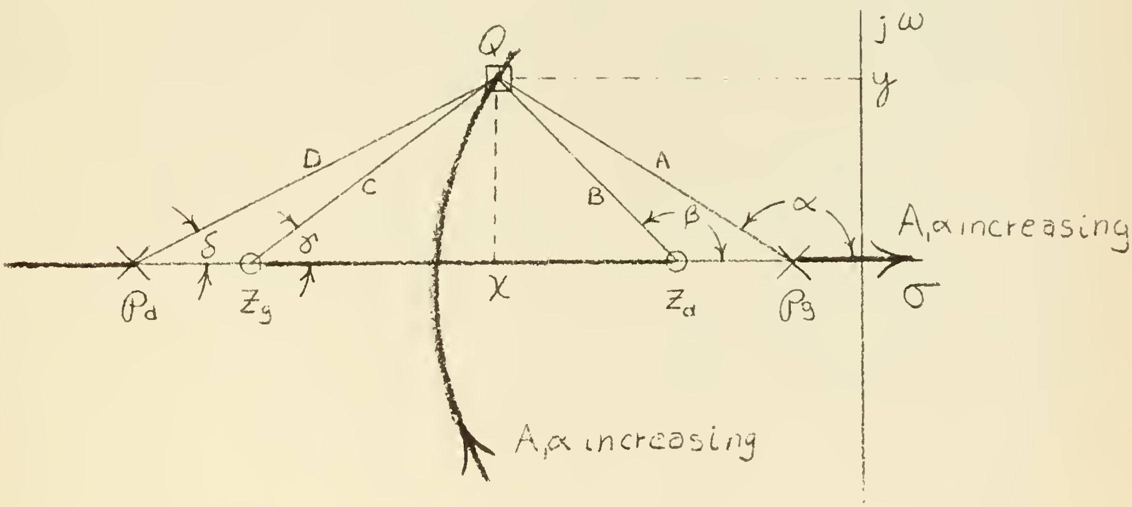

Fig. 3-34. A possible root locus of Equation 3.31.

This transfer function is of further interest because z_d and p_g, the real zeros of Equation 3.32 may be placed as close to the origin of the s-plane as desired. There can be no complex roots resulting from the root loci of Figs. 3-31. and 3-32. Therefore, they are of no further interest in this investigation.

3(O). Lag-lead positive feedback compensator: lead in the feedback path.

Fig. 3-35 is the block diagram of this compensator. The transfer function of Fig. 3-35 is

3.33 $$\frac{V_o}{V_i} = \frac{(s+z_g)(s+p_d)}{(s+p_g)(s+p_d) - A_1\alpha(s+z_d)(s+z_g)}$$

For factoring by root locus, the denominator of Equation 3.33 may be manipulated into the form

3.34 $$\frac{A_1\alpha(s+z_d)(s+z_g)}{(s+p_d)(s+p_g)} = 1$$

Equation 3.34 is identical to Equation 3.31. Therefore the root loci of Equation 3.34 will be those of Figs. 3-31, 3-32, 3-33, and 3-34. Thus, Equation 3.33 will be of the form

3.35 $$\frac{V_o}{V_i} = \frac{(s+z_g)(s+p_d)}{(1-A_1\alpha)(s+\sigma+j\omega)(s+\sigma-j\omega)}$$

The same comments may be made for Equation 3.35 as for Equation 3.32 except that z_g and p_d, the real zeros of Equation 3.35 may be placed far out in the left hand s-plane if desired.

3(P). Summary

As a result of the investigations of this section, the compensators of Sections 3(B), 3(F), and 3(I) will produce complex zeros. The compensators of Sections 3(C), 3(G), 3(N), and 3(O) will produce complex poles. It has been further shown that with those compensators producing

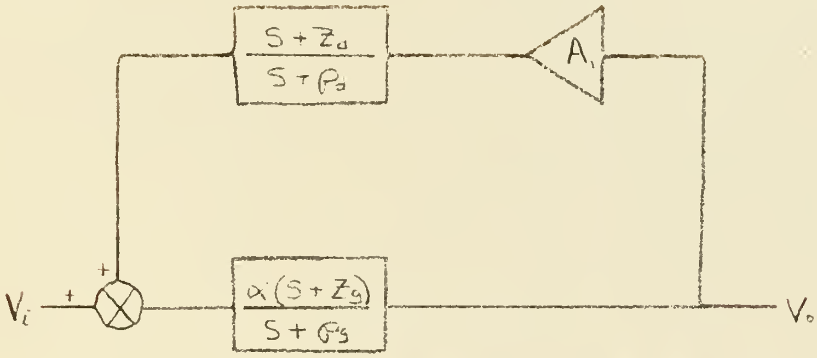

Fig. 3-35. Block diagram of the lag-lead positive feedback compensator. Lead in the feedback path.

complex zeros, there are always two real poles in addition to the zeros. These two real poles may be near the origin, one near the origin and one far out, or both far out in the left hand s-plane; depending on the particular active compensator chosen. For the compensators producing complex poles, there are always two real zeros in addition to the poles. These two real zeros may be near the origin, one near the origin and one far out, or both far out in the left hand s-plane; depending on the particular active compensator chosen.

The root loci of the complex poles and zeros are further investigated in Section 4.

4. Derivation of the Equations of the Root Loci of the Complex Poles and Zeros Resulting from the Networks of Section 3

It is now of interest to derive the equations of the root loci of the complex poles and zeros resulting from the compensators listed in Paragraph 3(P).

4(A). Lead-lead feed forward difference compensator.

Plotted on Figs. 3-4 and 3-5 are the root loci of Equation 3.4 which yield complex roots. On Fig. 3-5, let Q be a point with coordinates (x,y) which is on the root locus.

Since Equation 3.4 is a zero degree locus

$$\measuredangle Q = 2n\pi = -\alpha + \beta + \gamma - \delta$$

where $-\infty < n < \infty$ and is an integer.

4.1 $$\sin(\measuredangle Q) = 0 = \sin(-\alpha + \beta + \gamma - \delta)$$

Equation 4.1 may be expanded by trigonometric identities to obtain

4.2 $$\begin{aligned} 0 = & \left(\sin\beta\cos\alpha - \cos\beta\sin\alpha\right)\left(\cos\gamma\cos\delta + \sin\gamma\sin\delta\right) \\ & + \left(\cos\beta\cos\alpha + \sin\beta\sin\alpha\right)\left(\sin\gamma\cos\delta - \cos\gamma\sin\delta\right) \end{aligned}$$

From Fig. 3-5,

4.3
$$\sin\alpha = y/A \qquad \sin\gamma = y/C$$
$$\cos\alpha = \frac{x - z_{d2}}{A} \qquad \cos\gamma = \frac{x - P_{d2}}{C}$$
$$\sin\beta = y/B \qquad \sin\delta = y/D$$
$$\cos\beta = \frac{x - z_{d1}}{B} \qquad \cos\delta = \frac{x - P_{d1}}{D}$$

Substituting Equations 4.3 into Equation 4.2, expanding and collecting terms obtains

4.4 $$\left[x + \frac{P_{d2}z_{d1} - P_{d1}z_{d2}}{z_{d2} - z_{d1} + P_{d1} - P_{d2}}\right]^2 + y^2 = \frac{P_{d1}P_{d2}(z_{d1} - z_{d2}) + z_{d1}z_{d2}(P_{d2} - P_{d1})}{z_{d2} - z_{d1} + P_{d1} - P_{d2}} + \left[\frac{P_{d2}z_{d1} - P_{d1}z_{d2}}{z_{d2} - z_{d1} + P_{d1} - P_{d2}}\right]^2 + \frac{ABCD}{y}(0)$$

the equation of a circle.

This equation was derived with the $\measuredangle Q = 2n\pi$. Since identical results could have been obtained with $\measuredangle Q = (2n-1)\pi$, it must be shown that for this case,

$$\measuredangle Q \neq (2n-1)\pi$$

That this is the case may be seen from Fig. 3-5, since all of the $(2n-1)\pi$ loci are on the real axis. Equation 4.4 is not valid for y = 0. Therefore, in all areas for which Equation 4.4 holds, there are no $(2n-1)\pi$ loci. A similar argument may be applied to all derivations in this section.

On Fig. 3-4, let Q be a point with coordinates (x,y) which is on the root locus. Since Equation 3.4 is a zero degree locus

$$\measuredangle Q = 2n\pi = -\alpha + \beta + \gamma - \delta$$

where $-\infty < n < \infty$ and is an integer.

$$\sin \measuredangle Q = 0 = \sin(-\alpha + \beta + \gamma - \delta)$$

Since this is precisely Equation 4.1 and since Equations 4.3 also apply to Fig. 3.4, Equation 4.4 applies to Fig. 3-5 as well as Fig. 3-4.

4(B). Lag-lag feed forward difference compensator.

Plotted on Figs. 3-15 and 3-16 are the root loci of Equation 3.14 which yield complex roots. On Fig. 3-15, let Q be a point on the root locus with coordinates (x,y). Now since Equation 3.14 is a zero degree locus

$$\measuredangle Q = 2n\pi = -\alpha + \beta + \gamma - \delta$$

where $-\infty < n < \infty$ and is an integer.

But this is precisely Equation 4.1. The expansions and manipulations for this case will be identical to those of Section 4(A). The only difference

will be in the final result,

Equation 4.4 where

$$\rho_{g_1} \text{ replaces } Z_{d_2}$$

$$\rho_{g_2} \text{ replaces } Z_{d_1}$$

$$Z_{g_1} \text{ replaces } \rho_{d_2}$$

$$Z_{g_2} \text{ replaces } \rho_{d_1}$$

Making these substitutions in Equation 4.4 yields

$$4.5 \quad \left[x + \frac{\rho_{g_2} Z_{g_1} - \rho_{g_1} Z_{g_2}}{Z_{g_2} - Z_{g_1} + \rho_{g_1} - \rho_{g_2}}\right]^2 + y^2 = \frac{Z_{g_1} Z_{g_2}(\rho_{g_2} - \rho_{g_1}) + \rho_{g_1}\rho_{g_2}(Z_{g_1} - Z_{g_2})}{Z_{g_2} - Z_{g_1} + \rho_{g_1} - \rho_{g_2}} + \left[\frac{\rho_{g_2} Z_{g_1} - \rho_{g_1} Z_{g_2}}{Z_{g_2} - Z_{g_1} + \rho_{g_1} - \rho_{g_2}}\right]^2 + \frac{ABCD}{y}(0)$$

the equation of a circle.

On Fig. 3-16, let Q be a point with coordinates (x,y) which is on the root locus. Since Equation 3.14 is a zero degree locus

$$\measuredangle Q = 2n\pi = -\alpha + \beta + \gamma - \delta$$

Again, this is precisely Equation 4.1. Therefore, Equation 4.5 applies to Fig. 3-15 as well as Fig. 3.16.

4(C). Lag-lead feed forward summing compensator.

Plotted on Fig. 3-24 is the root locus of Equation 3.22. On Fig. 3-24 let Q be a point with coordinates (x,y) which is on the root locus. Now since Equation 3.24 is a 180 degree locus,

$$\measuredangle Q = (2n-1)\pi = -\alpha - \beta + \gamma + \delta$$

where $-\infty < n < \infty$ and is an integer

$$4.6 \qquad \sin \measuredangle Q = 0 = \sin\left[(\gamma + \delta) - (\alpha + \beta)\right]$$

Equation 4.6 may be expanded by trigonometric identities to obtain

$$0 = (\sin\gamma\cos\delta + \cos\gamma\sin\delta)(\sin\alpha\sin\beta - \cos\alpha\cos\beta)$$

4.7
$$+(\cos\gamma\cos\delta - \sin\gamma\sin\delta)(\sin\alpha\cos\beta + \cos\alpha\sin\beta)$$

From Fig. 3-22

4.8

$$\sin\alpha = y/A \qquad \sin\gamma = y/C$$

$$\cos\alpha = \frac{x-\rho_g}{A} \qquad \cos\gamma = \frac{x-z_g}{C}$$

$$\sin\beta = y/B \qquad \sin\delta = y/D$$

$$\cos\beta = \frac{x-z_d}{B} \qquad \cos\delta = \frac{x-\rho_d}{D}$$

Substituting Equations 4.8 into Equation 4.7, expanding and collecting terms yields

4.9
$$\left[x + \frac{\rho_d z_g - \rho_g z_d}{z_d - z_g + \rho_g - \rho_d}\right]^2 + y^2 = \frac{z_d z_g(\rho_d - \rho_g) + \rho_g\rho_d(z_g - z_d)}{z_d - z_g + \rho_g - \rho_d} + \left[\frac{\rho_d z_g - \rho_g z_d}{z_d - z_g + \rho_g - \rho_d}\right]^2 + \frac{ABCD}{y}(0)$$

the equation of a circle.

4(D). Lead-lead negative feedback compensator.

Plotted on Fig. 3-9 is the root locus of Equation 3.7. On Fig. 3-9, let Q be a point with coordinates (x,y) which is on the root locus. Now, since Equation 3.7 is a 180 degree root locus

$$\measuredangle Q = (2n-1)\pi = \alpha + \beta - \gamma - \delta$$

where $-\infty < n < \infty$ and is an integer.

4.10
$$\sin\measuredangle Q = 0 = \sin\left[(\alpha+\beta) - (\gamma+\delta)\right]$$

Equation 4.10 may be expanded by trigonometric identities to obtain

4.11
$$0 = (\sin\beta\cos\alpha + \cos\alpha\sin\beta)(\cos\gamma\cos\delta - \sin\gamma\sin\delta)$$
$$+ (\sin\alpha\sin\beta - \cos\alpha\cos\beta)(\sin\gamma\cos\delta + \cos\gamma\sin\delta)$$

From Fig. 3-9,

4.12
$$\sin\alpha = y/A \qquad \sin\gamma' = y/C$$
$$\cos\alpha = \frac{x - z_{d_1}}{A} \qquad \cos\gamma = \frac{x - \rho_{d_1}}{C}$$
$$\sin\beta = y/B \qquad \sin\delta = y/D$$
$$\cos\beta = \frac{x - z_{d_2}}{B} \qquad \cos\delta = \frac{x - \rho_{d_2}}{D}$$

Substituting Equations 4.12 into Equation 4.11, expanding and collecting terms obtains

4.13
$$\left[x + \frac{\rho_{d_1}\rho_{d_2} - z_{d_1}z_{d_2}}{z_{d_1} + z_{d_2} - \rho_{d_1} - \rho_{d_2}}\right]^2 + y^2 = \frac{\rho_{d_1}\rho_{d_2}(z_{d_1} + z_{d_2}) - z_{d_1}z_{d_2}(\rho_{d_1} + \rho_{d_2})}{z_{d_1} + z_{d_2} - \rho_{d_1} - \rho_{d_2}} + \left[\frac{\rho_{d_1}\rho_{d_2} - z_{d_1}z_{d_2}}{z_{d_1} + z_{d_2} - \rho_{d_1} - \rho_{d_2}}\right]^2 + \frac{ABCD}{y}(0)$$

the equation of a circle.

4(E). Lag-lag negative feedback compensator.

Plotted on Fig. 3-20 is the root locus of Equation 3.17. On Fig. 3-18, let Q be a point with coordinates (x,y) which is on the root locus. Now since Equation 3.14 is a 180 degree locus

$$\measuredangle Q = (2n - 1)\pi = -\alpha - \beta + \gamma + \delta$$

Where $-\infty < n < \infty$ and is an integer.

4.14
$$\sin\measuredangle Q = 0 = \sin\left[(\gamma + \delta) - (\alpha + \beta)\right]$$

This is precisely Equation 4.6. The expansions and manipulations for this case will be identical to those of Section 4(C). The only difference will be in the equation of the circle, Equation 4.9, where

ρ_{g_1} replaces ρ_g

ρ_{g_2} replaces z_d

z_{g_1} replaces z_g

z_{g_2} replaces ρ_d

Making these substitutions yields

$$4.15 \quad \left[x+\frac{z_{g_2}z_{g_1}-\rho_{g_2}\rho_{g_1}}{\rho_{g_1}+\rho_{g_2}-z_{g_1}-z_{g_2}}\right]^2+y^2=\frac{z_{g_1}z_{g_2}(\rho_{g_1}+\rho_{g_2})-\rho_{g_1}\rho_{g_2}(z_{g_1}+z_{g_2})}{\rho_{g_1}+\rho_{g_2}-z_{g_1}-z_{g_2}}+\left[\frac{z_{g_1}z_{g_2}-\rho_{g_1}\rho_{g_2}}{\rho_{g_1}+\rho_{g_2}-z_{g_1}-z_{g_2}}\right]^2+\frac{ABCD}{y}(0)$$

the equation of a circle.

4(F). Lag-lead positive feedback compensator.

Plotted on Figs. 3-33 and 3-34 are the root loci of Equation 3.31. On Fig. 3-33 let Q be a point with coordinates (x,y) which is on the root locus. Since Equation 3.31 is a zero degree locus

$$\measuredangle Q = 2n\pi = -\alpha+\beta+\gamma-\delta$$

where $-\infty < n < \infty$ and is an integer.

$$4.16 \quad \sin\measuredangle Q = 0 = \sin(-\alpha+\beta+\gamma-\delta)$$

Equation 4.16 may be expanded by trigonometric identities to obtain

$$4.17 \quad \begin{aligned} 0 &= (\sin\beta\cos\alpha-\cos\beta\sin\alpha)(\cos\gamma\cos\delta+\sin\gamma\sin\delta) \\ &+(\cos\beta\cos\alpha+\sin\beta\sin\alpha)(\sin\gamma\cos\delta-\cos\gamma\sin\delta) \end{aligned}$$

From Fig. 3-33

$$4.18 \quad \begin{aligned} \sin\alpha &= y/A & \sin\gamma &= y/C \\ \cos\alpha &= \frac{x-\rho_g}{A} & \cos\gamma &= \frac{x-z_g}{C} \\ \sin\beta &= y/B & \sin\delta &= y/D \\ \cos\beta &= \frac{x-z_d}{B} & \cos\delta &= \frac{x-\rho_d}{D} \end{aligned}$$

Substituting Equations 4.18 into Equation 4.17, expanding and collecting terms obtains

$$4.19 \quad \left[x+\frac{\rho_g\rho_d-z_gz_d}{z_g+z_d-\rho_g-\rho_d}\right]^2+y^2=\frac{\rho_d\rho_g(z_g+z_d)-z_gz_d(\rho_g+\rho_d)}{z_g+z_d-\rho_g-\rho_d}+\left[\frac{\rho_g\rho_d-z_gz_d}{z_g+z_d-\rho_g-\rho_d}\right]^2+\frac{ABCD}{y}(0)$$

the equation of a circle.

On Fig. 3-34 let Q be a point with coordinates (x,y) which is on the root locus. Since Equation 3.31 is a zero degree locus

$$\measuredangle Q = 2n\pi = -\alpha + \beta + \gamma - \delta$$

But this is precisely Equation 4.16. And since Equations 4.18 also apply to Fig. 3-34, Equation 4.19 must apply to Fig. 3-34 as well as Fig. 3-33.

4(C). Summary

These derivations prove that the equations for the root loci upon which the complex zeros or poles may lie are circles. The circles are completely specified by the poles and zeros of the passive components.

Table 4-1 is a compilation of the circular equations of complex zeros derived in this section. The similarity between the equations is obvious. All complex zero loci have the form

$$4.20 \quad \left[x + \frac{z_1 p_2 - z_2 p_1}{z_1 - z_2 + p_2 - p_1}\right]^2 + y^2 = \frac{z_1 z_2 (p_1 - p_2) + p_1 p_2 (z_2 - z_1)}{z_1 - z_2 + p_2 - p_1} + \left[\frac{z_1 p_2 - z_2 p_1}{z_1 - z_2 + p_2 - p_1}\right]^2$$

where z_1, z_2, p_1, and p_2 are the zeros and poles of lag or lead networks, depending on the particular active compensator chosen.

Table 4-2 is a compilation of the circular equations of complex poles derived in this section. Again, the similarity between the equations is obvious. All complex pole loci have the form

$$4.21 \quad \left[x + \frac{p_1 p_2 - z_1 z_2}{z_1 + z_2 - p_1 - p_2}\right]^2 + y^2 = \frac{p_1 p_2 (z_1 + z_2) - z_1 z_2 (p_1 + p_2)}{z_1 + z_2 - p_1 - p_2} + \left[\frac{p_1 p_2 - z_1 z_2}{z_1 + z_2 - p_1 - p_2}\right]^2$$

where z_1, z_2, p_1, and p_2 are the zeros and poles of lead or lag networks depending on the particular active compensator chosen.

COMPENSATOR	EQUATION OF THE ROOT LOCUS OF COMPLEX ZEROS
Lead-lead feed forward difference compensator	$\left[x + \frac{\rho_{d_2} z_{d_1} - \rho_{d_1} z_{d_2}}{z_{d_2} - z_{d_1} + \rho_{d_1} - \rho_{d_2}}\right]^2 + y^2 = \frac{\rho_{d_1}\rho_{d_2}(z_{d_1} - z_{d_2}) + z_{d_1} z_{d_2}(\rho_{d_2} - \rho_{d_1})}{z_{d_2} - z_{d_1} + \rho_{d_1} - \rho_{d_2}} + \left[\frac{\rho_{d_2} z_{d_1} - \rho_{d_1} z_{d_2}}{z_{d_2} - z_{d_1} + \rho_{d_1} - \rho_{d_2}}\right]^2$
Lag-lag feed forward difference compensator	$\left[x + \frac{\rho_{g_2} z_{g_1} - \rho_{g_1} z_{g_2}}{z_{g_2} - z_{g_1} + \rho_{g_1} - \rho_{g_2}}\right]^2 + y^2 = \frac{\rho_{g_1}\rho_{g_2}(z_{g_1} - z_{g_2}) + z_{g_1} z_{g_2}(\rho_{g_2} - \rho_{g_1})}{z_{g_2} - z_{g_1} + \rho_{g_1} - \rho_{g_2}} + \left[\frac{\rho_{g_2} z_{g_1} - \rho_{g_1} z_{g_2}}{z_{g_2} - z_{g_1} + \rho_{g_1} - \rho_{g_2}}\right]^2$
Lag-lead feed forward summing compensator	$\left[x + \frac{\rho_d z_g - \rho_g z_d}{-z_g + z_d + \rho_g - \rho_d}\right]^2 + y^2 = \frac{\rho_d \rho_g (z_g - z_d) + z_d z_g (\rho_d - \rho_g)}{z_d - z_g + \rho_g - \rho_d} + \left[\frac{\rho_d z_g - \rho_g z_d}{z_d - z_g + \rho_g - \rho_d}\right]^2$

TABLE 4-1

COMPENSATOR	EQUATION OF THE ROOT LOCUS OF COMPLEX POLES
Lead-lead negative feedback compensator	$\left[x + \frac{P_{d1}P_{d2} - Z_{d1}Z_{d2}}{Z_{d1}+Z_{d2} - P_{d1} - P_{d2}}\right]^2 + y^2 = \frac{P_{d1}P_{d2}(Z_{d1}+Z_{d2}) - Z_{d1}Z_{d2}(P_{d1}+P_{d2})}{Z_{d1}+Z_{d2} - P_{d1} - P_{d2}} + \left[\frac{P_{d1}P_{d2} - Z_{d1}Z_{d2}}{Z_{d1}+Z_{d2} - P_{d1} - P_{d2}}\right]^2$
Lag-lag negative feedback compensator	$\left[x + \frac{P_{g1}P_{g2} - Z_{g1}Z_{g2}}{Z_{g1}+Z_{g2} - P_{g1} - P_{g2}}\right]^2 + y^2 = \frac{P_{g1}P_{g2}(Z_{g1}+Z_{g2}) - Z_{g1}Z_{g2}(P_{g1}+P_{g2})}{Z_{g1}+Z_{g2} - P_{g1} - P_{g2}} + \left[\frac{P_{g1}P_{g2} - Z_{g1}Z_{g2}}{Z_{g1}+Z_{g2} - P_{g1} - P_{g2}}\right]^2$
Lag-lead positive feedback compensator	$\left[x + \frac{P_d P_g - Z_d Z_g}{Z_d + Z_g - P_d - P_g}\right]^2 + y^2 = \frac{P_g P_d(Z_d + Z_g) - Z_d Z_g(P_d + P_g)}{Z_d + Z_g - P_d - P_g} + \left[\frac{P_d P_g - Z_d Z_g}{Z_d + Z_g - P_d - P_g}\right]^2$

TABLE 4-2

Some further properties of these circular loci may be noted by inspection of Equations 4.20 and 4.21. The denominators of the terms which define the centers of the circles may approach zero more or less independently of the sign or magnitude of the numerator of those terms. Therefore the center of the circle may vary from negative to positive infinity. Similarly, the denominators of the terms which define the squared radii of the circles may approach zero more or less independently of the sign or the magnitude of the numerator. Thus it is possible to have radii which vary from zero to infinity. In the cases for which the squared radius is negative, of course, the circle is undefined. The denominators of the center and radii terms are the same, though, so as the center of the circle goes to infinity, the radius must also go to infinity. However, the center does not have to go to infinity at the same rate as the radius since the numerators of the terms are different. In fact, it is possible to have the center remain at the origin while the radius goes to infinity. The result of this is that in general, it is possible to locate complex poles or zeros anywhere in the left or right half of the s-plane, providing the correct compensator is chosen. Some of the circles are restricted to the left half s-plane by the nature of their root loci.

Preliminary investigations indicate that there are areas on the s-plane where complex poles or zeros may not be placed with a specific active compensator. These areas are obvious in some cases. In others, they are not only not obvious, but are obscured by the fact that their existance is a function of the four variables z_1 , z_2 , p_1 , and p_2 . All attempts to derive simple, general equations which would define these areas have met with failure.

To verify the existence of the circular root loci, an analog computer simulation of the lead-lead feed forward difference compensator and the lead-lead negative feedback compensator was performed. Appendix A contains the computer set up and test results. It was found that the locations of complex poles and zeros resulting from the previously mentioned networks could be predicted quite accurately. The agreement between the experimental and analytical responses of the circuits to a step input, as shown in Appendix A substantiate this.

5. Manipulations to Obtain Working Relationships for Designing of Active Compensators

The equations of the root locus circles, while of definite interest, are not suitable to work with when designing active compensators. A convenient form may be obtained by manipulation of Equation 4.20 for complex zeros and Equation 4.21 for complex poles.

5(A). Obtaining of a working relationship for complex zeros.

Equation 4.20 may be manipulated into the form

$$5.1 \quad (z_1 - z_2 + \rho_2 - \rho_1)(x^2 + y^2) + 2x(z_1\rho_2 - z_2\rho_1) + \rho_1\rho_2(z_1 - z_2) + z_1 z_2(\rho_2 - \rho_1) = 0$$

for use in design. Table 5-1 gives the particular form of Equation 5.1 applicable to each of the complex zero producing compensators. It will be shown in Section 7(A) that four of the six variables in Equation 5.1 may be chosen to meet the needs of a specific problem. Equation 5.1 will then reduce to an equation in two unknowns which may be chosen so as to satisfy that equation and the requirement of physical realizability.

A significant simplification of Equation 5.1 occurs if:

$$5.2 \quad \begin{aligned} z_1/\rho_1 &= z_2/\rho_2 \\ z_1\rho_2 &= z_2\rho_1 \triangleq K_1 \end{aligned}$$

Substituting Equation 5.2 into Equation 5.1 and simplifying yields

$$5.3 \quad x^2 + y^2 = K_1$$

Note that due to restrictions imposed by Equation 5.2, Equation 5.3 can be used only in those active compensators which contain two lag or two lead networks.

COMPENSATOR	WORKING EQUATION	NUMERATOR ROOT LOCUS EQUATION	REAL POLES	K_c
Lead-lead feed forward difference compensator	$(z_{d_2} - z_{d_1} + \rho_{d_1} - \rho_{d_2})(x^2 + y^2) + 2x(\rho_{d_2} z_{d_1} - \rho_{d_1} z_{d_2}) = \rho_{d_1}\rho_{d_2}(z_{d_1} - z_{d_2}) + z_{d_1} z_{d_2}(\rho_{d_2} - \rho_{d_1})$	$\frac{A_1(s+z_{d_1})(s+\rho_{d_2})}{A_2(s+z_{d_2})(s+\rho_{d_1})} = 1$	ρ_{d_1}, ρ_{d_2}	$(A_1 - A_2)$
Lag-lag feed forward difference compensator	$(z_{g_2} - z_{g_1} + \rho_{g_1} - \rho_{g_2})(x^2 + y^2) + 2x(\rho_{g_2} z_{g_1} - \rho_{g_1} z_{g_2}) = \rho_{g_1}\rho_{g_2}(z_{g_1} - z_{g_2}) + z_{g_1} z_{g_2}(\rho_{g_2} - \rho_{g_1})$	$\frac{A_1\alpha_1(s+z_{g_1})(s+\rho_{g_2})}{A_2\alpha_2(s+z_{g_2})(s+\rho_{g_1})} = 1$	ρ_{g_1}, ρ_{g_2}	$(A_1\alpha_1 - A_2\alpha_2)$
Lag-lead feed forward summing compensator.	$(z_d - z_g + \rho_g - \rho_d)(x^2 + y^2) + 2x(\rho_d z_g - \rho_g z_d) = z_d z_g(\rho_d - \rho_g) + \rho_d \rho_g(z_g - z_d)$	$\frac{A_1\alpha(s+z_g)(s+\rho_d)}{A_2(s+z_d)(s+\rho_g)} = -1$	ρ_d, ρ_g	$(A_1\alpha + A_2)$

TABLE 5-1

5(E). Obtaining of a working relationship for complex poles.

Equation 4.11 may be manipulated into the form

$$5.4 \quad (z_1 + z_2 - \rho_1 - \rho_2)(x^2 + y^2) + 2x(\rho_1\rho_2 - z_1 z_2) + z_1 z_2(\rho_1 + \rho_2) - \rho_1\rho_2(z_1 + z_2) = 0$$

for use in design. Table 5-1 gives the particular form of Equation 5.4 applicable to each of the complex pole producing compensators. It will be shown in Section 7(B) that four of the six variables in Equation 5.4 may be chosen to meet the needs of a specific problem. Equation 5.4 will then reduce to an equation in two unknowns which may be chosen so as to satisfy that equation and the requirement of physical realizability.

A significant simplification of Equation 5.4 occurs if

$$5.5 \qquad z_1 z_2 = \rho_1 \rho_2 \triangleq K_2$$

Substituting Equation 5.5 into Equation 5.4 and simplifying yields

$$5.6 \qquad x^2 + y^2 = K_2$$

Note that due to restrictions imposed by Equation 5.5, Equation 5.6 can be used only in those active compensators which contain a lag and a lead network.

COMPENSATOR	WORKING EQUATION	DENOMINATOR ROOT LOCUS EQUATION	REAL ZEROS	K_c
Lead-lead negative feedback compensator	$(Z_{d_1}+Z_{d_2}-P_{d_1}-P_{d_2})(x^2+y^2)+2x(P_{d_1}P_{d_2}-Z_{d_1}Z_{d_2}) = P_{d_1}P_{d_2}(Z_{d_1}+Z_{d_2})-Z_{d_1}Z_{d_2}(P_{d_1}+P_{d_2})$	$\frac{A_1(s+Z_{d_1})(s+Z_{d_2})}{(s+P_{d_1})(s+P_{d_2})} = -1$	Z_{d_1}, P_{d_2}	$\frac{1}{1+A_1}$
Lag-lag negative feedback compensator	$(Z_{g_1}+Z_{g_2}-P_{g_1}-P_{g_2})(x^2+y^2)+2x(P_{g_1}P_{g_2}-Z_{g_1}Z_{g_2}) = P_{g_1}P_{g_2}(Z_{g_1}+Z_{g_2})-Z_{g_1}Z_{g_2}(P_{g_1}+P_{g_2})$	$\frac{A_1\alpha_1\alpha_2(s+Z_{g_1})(s+Z_{g_2})}{(s+P_{g_1})(s+P_{g_2})} = -1$	Z_{g_1}, P_{g_2}	$\frac{1}{1+A_1\alpha_1\alpha_2}$
Lag-lead positive feedback compensator, lag in feedback path	$(Z_g+Z_d-P_g-P_d)(x^2+y^2)+2x(P_gP_d-Z_gZ_d) = P_gP_d(Z_d+Z_g)-Z_gZ_d(P_g+P_d)$	$\frac{A_1\alpha(s+Z_g)(s+Z_d)}{(s+P_g)(s+P_d)} = 1$	Z_g, P_d	$\frac{1}{1-A_1\alpha}$
Lag-lead positive feedback compensator, lead in feedback path	$(Z_g+Z_d-P_g-P_d)(x^2+y^2)+2x(P_gP_d-Z_gZ_d) = P_gP_d(Z_d+Z_g)-Z_gZ_d(P_d+P_g)$	$\frac{A_1\alpha(s+Z_g)(s+Z_d)}{(s+P_g)(s+P_d)} = 1$	P_g, Z_d	$\frac{1}{1-A_1\alpha}$

TABLE 5-2

6. Construction of the Locus of Complex Poles or Complex Zeros Producing a Constant Phase Angle at a Point in the S-plane.

It has been shown by Carpenter [4] that if the angle at a point in the s-plane due to an array of poles and zeros be known, it is possible to construct the locus of complex conjugate poles or complex conjugate zeros which will cause this point to be on a root locus. This locus of complex poles or zeros proves to be an arc of a circle. Carpenter's derivation is as follows:

> Figure 6-1 shows the necessary construction in proving the locus of complex conjugate poles or zeros, R and R', producing a constant phase at a point P, in the s-plane, is an arc of a circle passing through that point. The center of the circle lies on the real axis.
>
> From the figure:
>
> $$a = 90^\circ + \alpha + \beta$$
>
> $$b = 90^\circ + \alpha$$
>
> 6.1 $a + b = \beta + 2\alpha + 180 = N$ = angle contributed by complex pair at point P
>
> 6.2 But $\beta + 2\alpha + \gamma = 180^\circ$
> Subtracting 6.2 from 6.1
>
> $$360 - N = \gamma = \text{a constant}$$
>
> The vertex of γ, therefore, lies on an arc PP', of a circle whose center is on the real axis, by a fundamental theorem of plane geometry.
>
> Since the arc always intersects the real axis, the arc may be quickly constructed by first locating that intersection. The intersection is found by constructing the angle,
>
> 6.3 $\phi = N/2$
>
> at the point P.

It can also be shown that the construction angle at the point P which locates the center of the circular arc on the real axis is

$$\Gamma = N$$

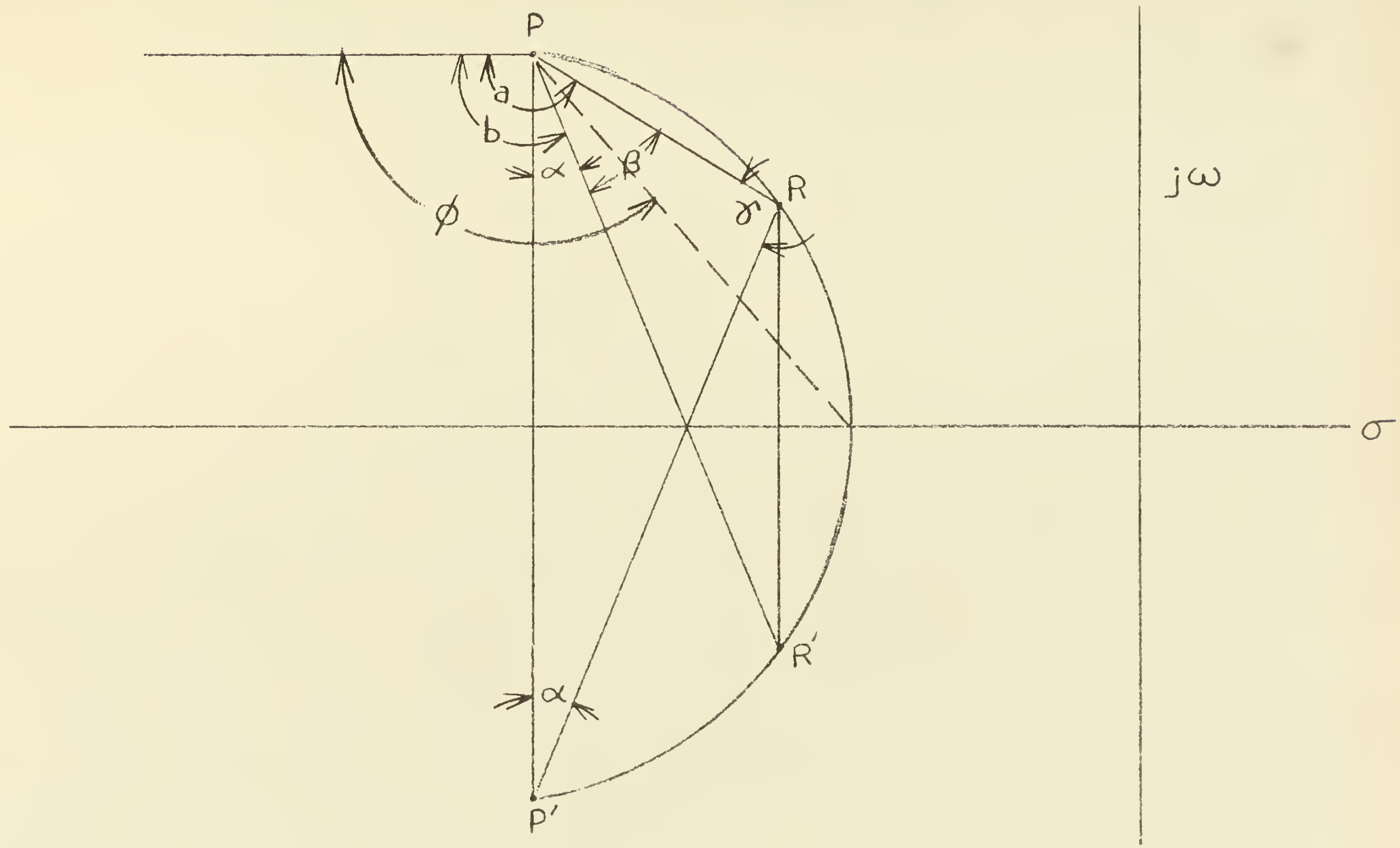

Fig. 6-1. Construction necessary in proving Carpenter's work.

The necessary construction to show this is Fig. 6-2. F is the intersection of the circular arc with the real axis defined by the angle ϕ. FG is the perpendicular bisector of PF. Since the center of the circular arc is on the real axis, the intersection G must be the center of the circular arc, by plane geometry. From Fig. 6-2:

$$\phi + g = 90^\circ$$

6.4 $$2\phi + 2g = 180^\circ$$

6.5 $$2g = h$$

Subtracting Equation 6.5 from 6.4 obtains

6.6 $$2\phi = 180^\circ - h$$

$$h + \Gamma = 180^\circ$$

6.7 $$\Gamma = 180^\circ - h$$

Subtracting Equation 6.7 from 6.6

$$2\phi = \Gamma = H$$

from Equation 6.5, thus proving the statement.

With the center of the circular arc and its point of intersection with the real axis known, from ϕ and K, the arc may be easily constructed.

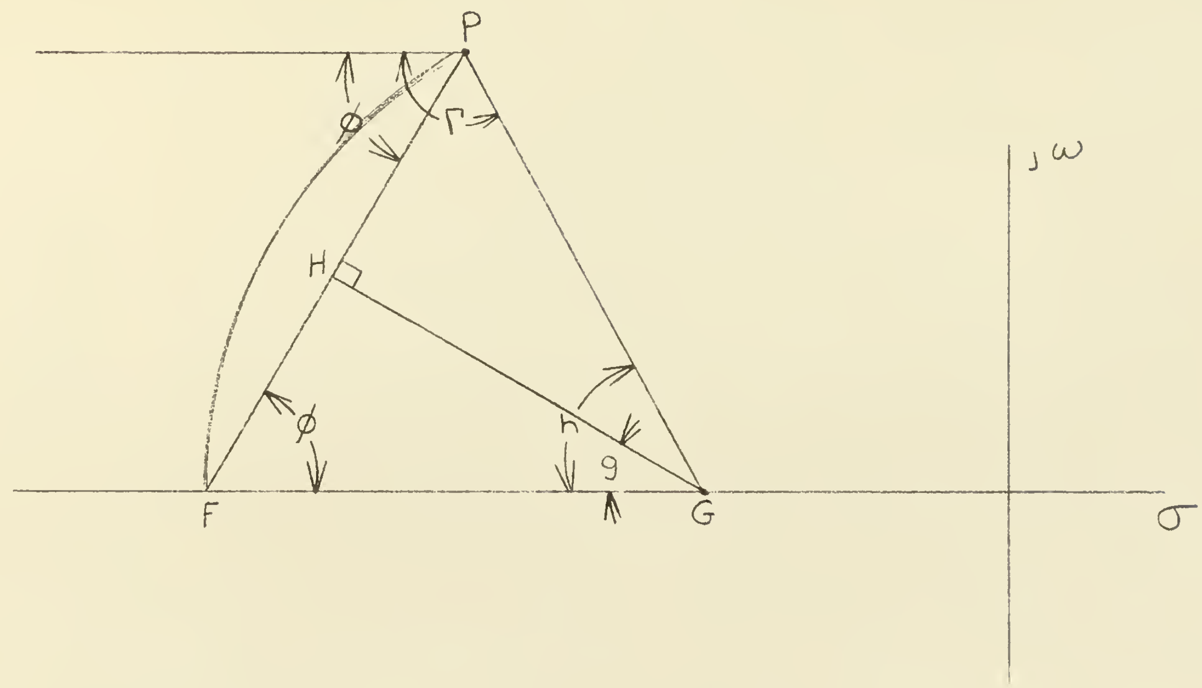

Fig. 6-2. Construction necessary in locating the center of the circular arc

7. A Technique for Compensation Using Active Networks

Relocation of the root loci of the characteristic equation so that they pass through a specified point is not in itself any great problem. Simultaneously meeting the requirements of dominance, steady state accuracy, and transient response is, however, quite another matter. A considerable number of techniques exist for the simultaneous solution of the steady state accuracy and root location problems. See for example [6, 7]. These techniques apply only to compensation with negative real poles and zeros and do not easily extend to the use of complex poles and zeros. The technique presented here requires some intuition and trial and error as thus far no more straight forward approach has been found. For the trial and error phases, the Electro-measurements, Inc., ESIAC algebraic Computer, Model 10 was a valuable tool.

The techniques of solution, whether for complex poles or complex zeros, are practically identical. Therefore they will be presented separately to avoid any possibility of confusion.

7(A). Solutions requiring complex zeros.

a. Select the compensation path. Then determine the open loop transfer function and manipulate the characteristic equation into the form given by Equation 1.1.

$$\frac{K\prod_{i=1}^{n}(s+z_i)}{s^m\prod_{j=1}^{r}(s+p_j)}G_c = -1$$

b. Plot the z_i, p_j, and s^m on the s-plane.

c. Select the real poles of G_c so as to be most convenient. Convenience will be determined by the particular problem. Plot these poles on the s-plane. This will determine which of the complex zero producing active networks is to be used.

d. Select the desired locations, P and P', of two complex conjugate closed loop roots. Plot P and P' on the s-plane. As mentioned previously, it is hoped that these roots will prove dominant, but there is no way of ensuring this.

e. Calculate the phase angle at P or P' due to z_i, p_j, s^m, and the real poles of G_c. Then construct the locus of complex zeros, in the manner described in Section 6, which will result in P and P' being on a 180 degree locus.

f. Choose two complex conjugate points, Q and Q', on this constructed locus. These will be the complex zeros of G_c.

g. Determine the root locus gain, K_{RL} which will cause roots to be at P and P' with the z_i, p_j, s^m, and the chosen real poles and complex zeros of G_c.

h. Substitute K_{RL} and the real poles and complex zeros of G_c into the open loop transfer function to determine if steady state accuracy specifications are met. If not, choose two new locations of the complex zeros of G_c and repeat steps f and g.

i. Determine x and y from the chosen locations of the complex zeros of G_c.

j. Substitute x and y and the selected real poles of G_c into the appropriate equation of Table 5-1 to determine the algebraic relationship between the real zeros of the compensator passive networks.

k. Construct a graph of this equation or evaluate it for several values of one variable and select any convenient, physically realizable values for the zeros of the passive networks. The z/p ratios of lead networks should be ≥ 0.1. [5]

l. Substitute the real poles and zeros of the passive networks into the numerator root locus equation of Table 5-1 corresponding to the

active network chosen. Plot this equation and the selected complex compensator zeros on another s-plane.

m. Determine the value of A_1/A_2 which will cause the roots of the numerator of G_c to lie at the selected locations for the complex zeros.

n. Using the results of steps g and m, A_1 and A_2 may be calculated and the compensator design is completed.

o. The transient response must be computed and compliance with specifications determined.

7(B). Solutions requiring complex poles.

a. Select the compensation path. Then determine the open loop transfer function and manipulate the characteristic equation into the form given by Equation 1.1.

$$\frac{K \prod_{i=1}^{r} (s + z_i)}{s^m \prod_{j=1}^{n} (s + p_j)} G_c = -1$$

b. Plot the z_i , p_j , and s^m on the s-plane.

c. Select the real zeros of G_c so as to be most convenient. Convenience will be determined by the particular problem. Plot these zeros on the s-plane. This selection will determine the particular active network to be used.

d. Select the desired locations, P and P', of two complex conjugate closed loop roots. Plot P and P' on the s-plane. As mentioned previously, it is hoped that these roots will be dominant, but there is no way of ensuring this.

e. Calculate the phase angle at P or P' due to z_i , p_j , s^m, and the real zeros of G_c. Then construct the locus of complex poles, in the manner described in Section 6, which will result in P and P' being

on a 180 degree locus.

f. Choose two complex conjugate points on this constructed locus. These will be the complex poles of G_c.

g. Determine the root locus gain, K_{RL}, which will cause roots to be at P and P' with the z_i , p_j , s^m , and the chosen complex poles and real zeros of G_c.

h. Substitute K_{RL} and the complex poles and real zeros of G_c into the open loop transfer function of the system to determine if steady state accuracy specifications are met. If not, choose new locations for the complex poles of G_c and repeat steps f and g.

i. Determine x and y from the chosen locations of the complex poles of G .

j. Substitute x and y and the selected real zeros of G_c into the appropriate equation of Table 5-2 to find the algebraic relationship between the real poles of the passive networks.

k. Construct a graph of this equation or evaluate it for several values of one variable and select any convenient, physically realizable values of the passive network poles. The z/p ratio for lead networks should be ≥ 0.1. [5]

l. Substitute the values of the poles and zeros of the passive networks into the denominator root locus equation of Table 5-2 corresponding to the active network chosen. Plot this equation and the selected complex compensating poles on another s-plane.

m. Determine the value of A_1 which will cause the roots of the denominator of G_c to lie at the selected locations of the complex compensating poles. This completes the design of the compensator.

n. The transient response must now be computed and compliance with specifications determined.

8. Numerical Examples

This section applies the techniques of solution presented in Section 7 to further clarify their use. The examples presented herein are straightforward so as to illustrate the use of active networks as compensators without becoming unnecessarily involved in other considerations. It is assumed that active compensation was necessary in these examples due to unspecified considerations. The subsections of each numerical example correspond to the steps in the applicapable technique of solution in Section 7.

8(A). Example 1

<u>Given:</u>

The feedback control system of Fig. 8-1. $K_v = 5.0$.

<u>Requirements:</u>

Use cascade compensation. Desired root location to be such that $0.5 \leq \zeta \leq 0.7$, settling time = 1.0 seconds. K_v not to be reduced.

<u>Solution:</u>

a. The compensated open loop transfer function of Fig. 8-1 is

$$F_c = \frac{K}{s(s^2+5s+100)} G_c$$

The characteristic equation of the compensated system is

8.1 $$1 + \frac{K}{s(s^2+5s+100)} = 0$$

b. The z_i, p_j, and s^m of Equation 8.1 are plotted on Fig. 8-2.

c. Select $p_{d_1} = -25$, $p_{d_2} = -30$, so as not to add any more poles in the region near the origin. p_{d_1} and p_{d_2} are also plotted on Fig. 8-2.

d. Select the points $s = -5 \pm j\,7$ to be P and P', the roots of the characteristic equation which will meet the requirements.

e. From Fig. 8-2, the lead angle required at P is 307.3 degrees.

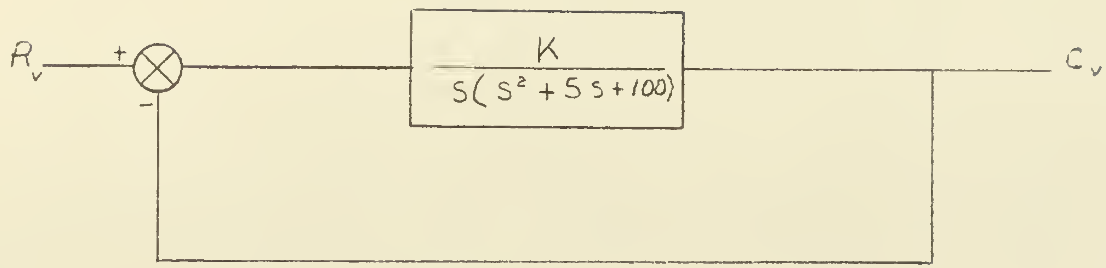

Fig. C-1. Block diagram of the feedback control system for Example 1.

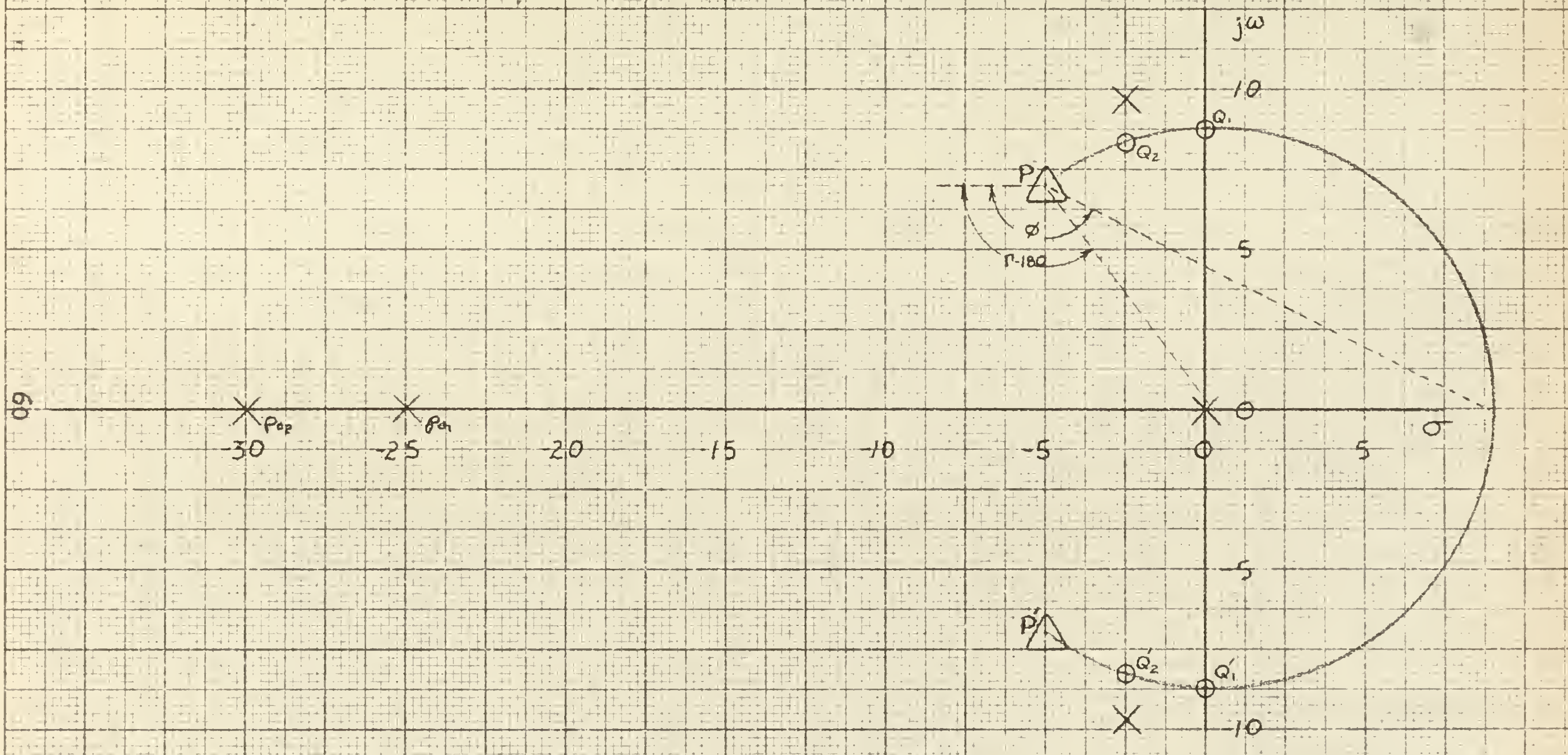

Fig. 8-2. S-plane constructions for solving Example 1.

The construction angles at P are

$$\phi = 155.65 \text{ degrees}$$

$$\Gamma = 307.3 \text{ degrees}$$

The arc of the circle on which complex compensating zeros can be located was constructed on Fig. 8-2 using these angles.

f. On Fig. 8-2, choose $s = \pm j\,8.8$ to be Q_1 and Q_1', the locations of the complex compensator zeros.

g. The root locus gain at P using all poles and zeros is

$$K_{RL} = 3,250$$

h. Substituting this value of gain and the chosen compensator poles and zeros into the open loop transfer function obtains $K_v = 3.51$. This does not meet requirements. Therefore, choose two new complex compensator zeros a little closer to the desired root locations. Let these new locations be $s = -2.5 \pm j\,8.4$, Q_2 and Q_2'. With these new zeros, the root locus gain at P is

$$K_{RL} = 6,578$$

Substituting this root locus gain and the new complex zeros into the open loop transfer function yields $K_v = 6.72$. This meets the requirements.

i. From the chosen locations of the complex zeros,

$$x = -2.5$$

$$y = \pm 8.4$$

j. Substituting these values of x and y, p_{d_1} and p_{d_2} into the working equation of the lead-lead feed forward difference compensator of Table 5-1 yields

8.2 $$z_{d_1} = 140.34 \left[\frac{z_{d_2} + .5466}{135.34 - z_{d_2}}\right]$$

The algebraic relationship between z_{d_1} and z_{d_2}.

k. Several values of z_{d_2} were substituted into this equation and $z_{d_1} = -2.5$, $z_{d_2} = -3.0$ were selected as the zeros of the lead networks.

l. Substituting the chosen z_{d_1}, z_{d_2}, p_{d_1}, and p_{d_2} into the numerator root locus equation of the lead-lead feed forward difference compensator of Table 5-1 yields

8.3 $$\frac{A_1 (s+2.5)(s+30)}{A_2(s+3)(s+25)} = 1$$

Equation 8.3 and the chosen complex compensator zeros are plotted on Fig. 8-3.

m. From Fig. 8-3, $A_1/A_2 = 0.8368$ will cause the numerator zeros to be at the chosen complex conjugate locations.

n. The values of A_1 and A_2 may now be computed from the equations

8.4 $$A_1/A_2 = 0.8368$$
$$K(A_1 - A_2) = K_{RL} = 6578$$

Since K is a variable gain element, any values of K, A_1, and A_2, which satisfy Equations 8.4 will yield satisfactory results.

o. The characteristic equation of the compensated system may now be written as

$$\frac{6578(s+2.5+j8.4)(s+2.5-j8.4)}{s(s^2+10s+100)(s+25)(s+30)} = -1$$

The roots of this equation were obtained by calculation on the Control Data Corporation 1604 digital computer as

$$s = -5.11 + j\,7.26$$
$$s = -5.11 - j\,7.26$$
$$s = -4.69 + j\,11.69$$
$$s = -4.69 - j\,11.69$$
$$s = -40.4$$

The roots as $s = 5.11 + j\,7.26$ and $s = -5.11 - j\,7.26$ correspond to those selected early in the solution. There is some error due to the graphical techniques required in the solution, however the agreement is close enough

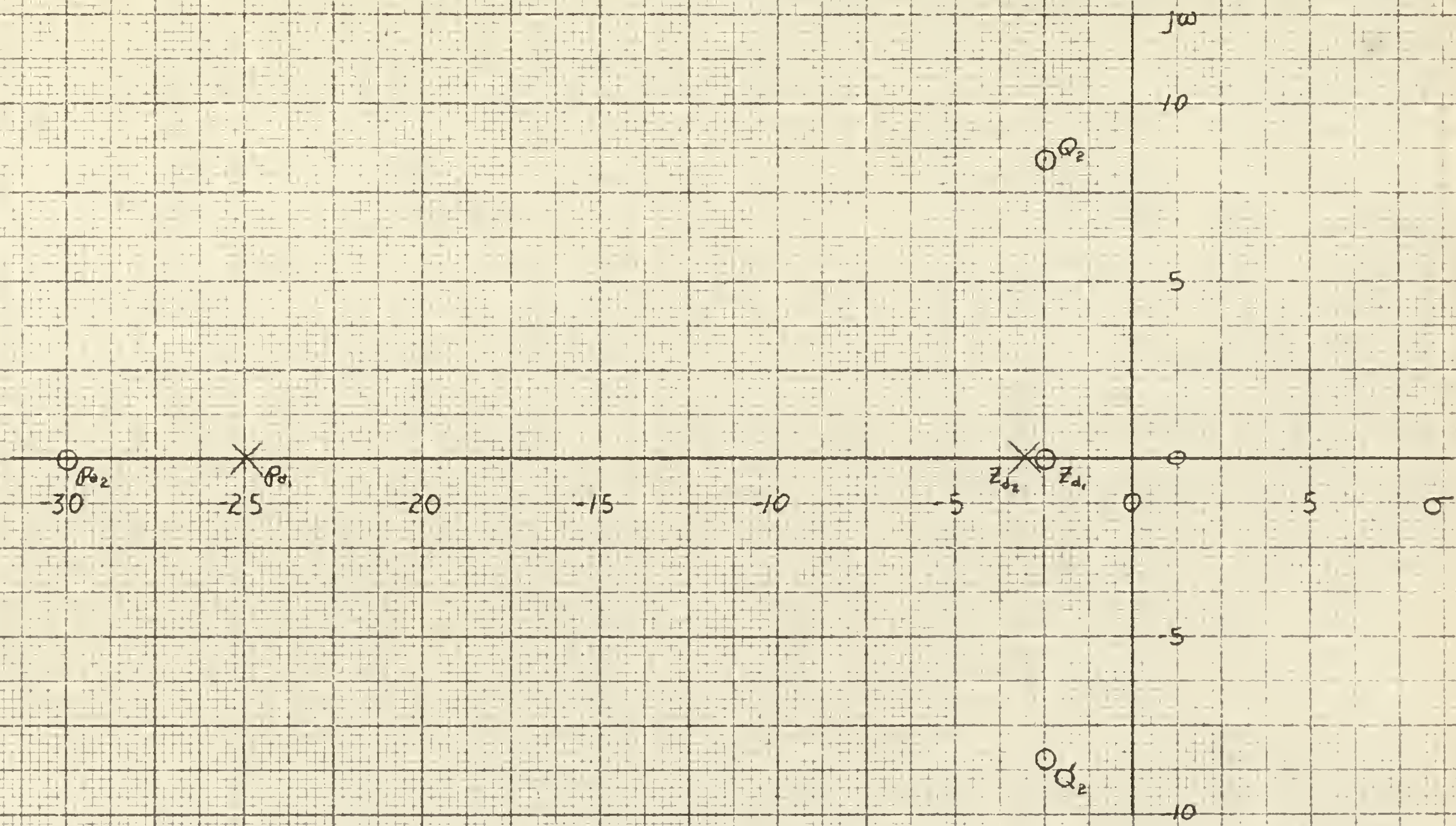

Fig. 8-3. S-plane pole zero array of Equation 8.3.

for engineering work. The transient response may now be calculated and final agreement with requirements determined.

8(B). Example 2.

Given:

The feedback control system of Fig. 8-4. $K_v = 100$

Requirements:

Use cascade compensation. Desired root location such that $0.5 \leq \zeta \leq 0.7$, settling time = 1.0 seconds. K_v not to be reduced.

Solution:

a. The compensated open loop transfer function of Fig. 8-4 is

$$F_o = \frac{K}{s(s+1)^2} G_c$$

The characteristic equation of the compensated system is

8.5 $$1 + \frac{K}{s(s+1)^2} G_c = 0$$

b. The z_i, p_j, and s^m of Equation 8.4 are plotted on Fig. 8.5.

c. Select $p_{d_1} = -40$, $p_{d_2} = -50$ so any root loci from these poles will not affect the region where the desired root will be located. p_{d_1} and p_{d_2} are also plotted on Fig. 8-5.

d. Select the points $s = -4 \pm j\,5$ to be P and P', the roots of the characteristic equation which will satisfy the requirements.

e. From Fig. 8-5, the lead angle required at P is 203.8 degrees. The construction angles at P are

$$\phi = 101.9 \text{ degrees}$$

$$\Gamma = 201.8 \text{ degrees}$$

The arc of the circle on which complex compensating zeros can be located was constructed on Fig. 8-5 from these angles.

f. Choose the points $s = -3.2 \pm j\,2$ to be Q and Q', the locations

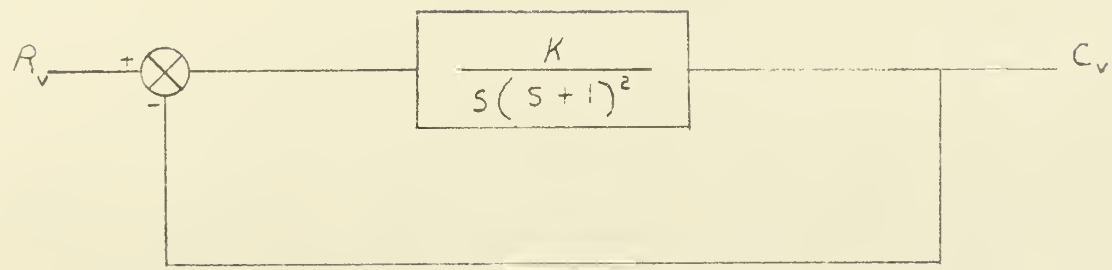

Fig. [illegible]. Block diagram of the feedback control system for Example 2.

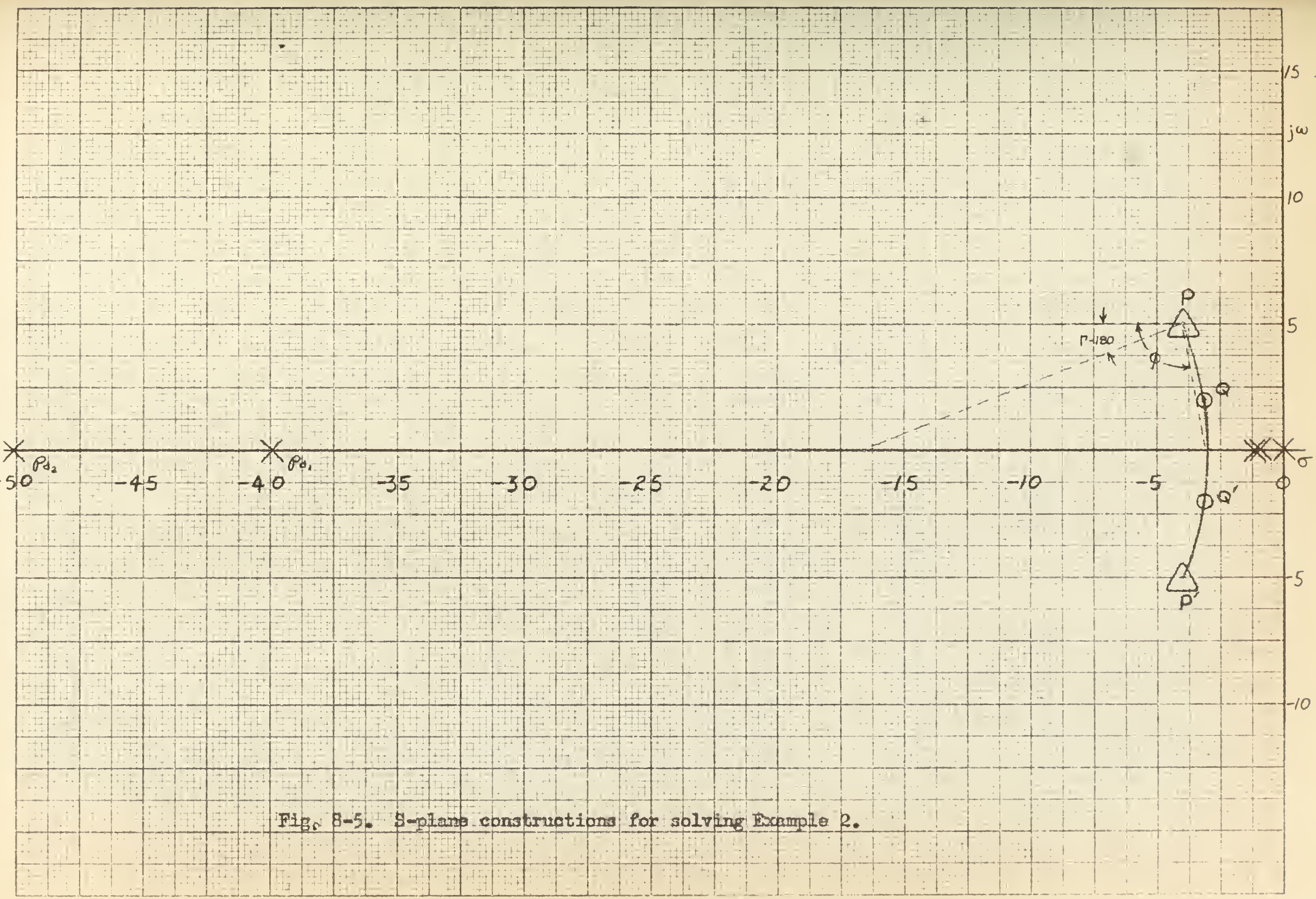

Fig. 8-5. S-plane constructions for solving Example 2.

of the complex compensator zeros.

g. The root locus gain at P and P' using all poles and zeros is

$$K_{PL} = 15,316$$

h. Substituting this value of gain and the chosen poles and complex zeros of the compensator into the open loop transfer function obtains $K_v = 108.8$.

This meets requirements.

i. From the chosen locations of the complex compensating zeros:

$$x = -3.2$$

$$y = \pm 2.0$$

j. Substituting these values of x and y, p_{d_1} and p_{d_2} into the working equation of the lead-lead feed forward difference compensator from Table 5-1 yields

$$z_{d_2} = 169.424 \left[\frac{z_{d_1} - .08405}{z_{d_1} + 175.824}\right]$$

the algebraic relationship between z_{d_1} and z_{d_2}.

k. Several values of z_{d_1} were substituted into this equation and $z_{d_1} = -30$, $z_{d_2} = -34.95$ were selected as the zeros of the lead networks.

l. Substituting the chosen z_{d_1}, z_{d_2}, p_{d_1}, and p_{d_2} into the numerator root locus equation of the lead-lead feed forward difference compensator on Table 5-1 yields

8.6 $$\frac{A_1(s+30)(s+50)}{A_2(s+34.95)(s+40)} = 1$$

Equation 8.6 and the chosen complex compensator zeros are plotted on Fig. 8-6.

m. From Fig. 8-6 $A_1/A_2 = 0.9314$ will cause the numerator zeros to be at the chosen complex conjugate locations.

n. The values of A_1 and A_2 may now be computed from the equations

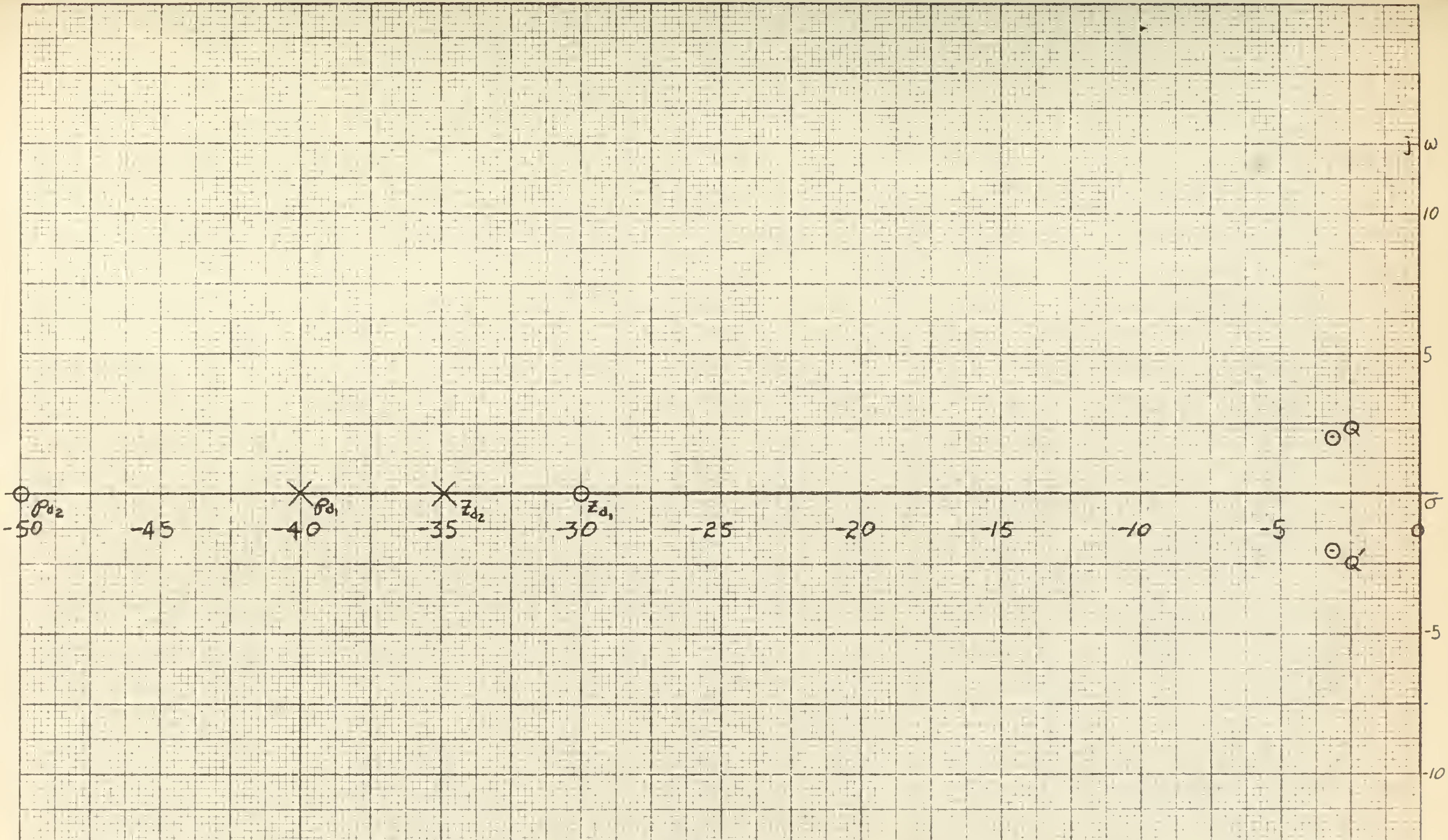

Fig. 8-6. S-plane pole zero array of Equation 8.6.

$$A_1/A_2 = 0.9314$$

8.7
$$K(A_1 - A_2) = K_{RL} = 15,316$$

Since K is a variable gain element, any values of K, A_1, and A_2, which satisfy Equations 8.7 will yield satisfactory results.

o. The characteristic equation of the compensated system may now be written as

$$\frac{15,316\,(s+3.2+j2)(s+3.2-j2)}{s\,(s+1)^2\,(s+40)(s+50)} = -1$$

The roots of this equation were obtained by calculation on the Control Data Corporation 1604 digital computer as

$$\begin{aligned} s &= -3.39 + j\,5.4 \\ s &= -3.39 - j\,5.4 \\ s &= -4.33 \\ s &= -19.73 \\ s &= -61.06 \end{aligned}$$

The roots as $s = -3.39 + j\,5.4$ and $s = -3.39 - j\,5.4$ correspond to those selected early in the solution. There is some error due to the graphical techniques required in the solution, however the agreement is close enough for engineering work. The transient response may now be calculated and final agreement with the requirements determined.

8(C). Summary

For both the examples chosen, the lead-lead feed forward difference compensator proved to be the most practical for solution. However, the use of the other active networks follows exactly the same procedures as demonstrated.

It was found that there are some regions on the circular loci of the active networks at which the gain ratio A_1/A_2 is extremely critical; i.e., a small variation in the gain ratio results in a large movement of the complex zeros of the active network. This sensitivity may be overcome

to some extent by a trial and error method of varying the real poles and zeros of the passive networks so that the critical area of the circular locus is moved away from the desired location of the complex zeros.

9. Conclusions and Recommendations

The active networks presented in this thesis afforded a simple method of generating complex poles and zeros. The locations of these complex poles and zeros may be accurately predicted and are easily varied by adjusting an amplifier gain or one or more of the passive elements of the networks. The varying of an amplifier gain is the more attractive of these two methods as it results in the complex poles or zeros moving along a circle which is precisely defined by the passive circuit elements.

It is highly probable that there exist other simple active networks which will produce complex poles and zeros. Further research is needed to determine those networks and investigate their characteristics.

Since the location of the complex poles and zeros is a function of amplifier gain, a requirement for a very stable amplifier is generated. This requirement may be relaxed somewhat if the compensated system is not too sensitive to the compensator pole and zero locations.

The lag-lead feed forward summing compensator has characteristics which merit special attention. Since the K_c of this compensator is $A_1 + A_2$, the possibility presents itself of using this compensator as the main transmission path amplifier. The complex zeros resulting from the network could be adjusted to best suit the needs of a specific system while the overall gain remained independent.

The technique presented in Section 7 for the use of these networks in solving compensation problems require considerable intuitive reasoning. It is recommended that further investigation be conducted to determine a more mathematically sound method of using these active networks as compensators.

In the numerical examples, it was necessary to assume that the

requirement for active compensation had been necessitated by some unspecified criteria. It is recommended that research be undertaken to determine generalized criteria which will indicate whether a specific problem should be attempted with active or passive compensation; and further if active compensation is desirable, which active network is best suited to the compensation problem to be solved.

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6. E. R. Ross and T. C. Warren, An Exact Method of Servomechanism Compensation Developed from a Study of Phase Angle and Gain Loci, Master's Thesis, United States Naval Postgraduate School, Monterey, California, 1959

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Equipment

The analog computer used in this experiment was The Donner Scientific Company, Model 3000. This computer is a semi-portable, ten amplifier, unstabilized, relatively inexpensive computer which is generally used for elementary classroom and laboratory work. The computer has reasonably good characteristics for its size and cost. Each operational amplifier is a stable, high gain D. C. circuit, with a pentode driving a cathode follower. Average gain is more than 10,000. Input impedance is that of an open grid pentode, and the output impedance less than one ohm. Long term drift is -4 mv/hr. Output range is -100 to +100 volts with load currents up to 5 ma.

All responses were recorded using the Brush Electronics Company Model 550 high gain direct coupled amplifier and BL-202 Oscillograph.

Lead-lead feed forward difference compensator

This compensator is simulated by the circuit shown on Fig. A-1. This experiment proves the results of Equation 4.4 in that given two lead networks, a circle upon which complex zeros may be located is specified. The exact location of the zeros is fixed by the gain ratio A_1/A_2. Using the lead networks

$$G_{d_1}(s) = \frac{s+0.5}{s+0.3}$$

$$G_{d_2}(s) = \frac{s+0.1}{s+0.6}$$

a circle with center, c = 0, and radius, r = 0.548 is specified by Equation 4.4. The numerator root locus equation from Table 5-1 is

$$\frac{A_1(s+Z_{d_1})(s+P_{d_2})}{A_2(s+Z_{d_2})(s+P_{d_1})} = 1$$

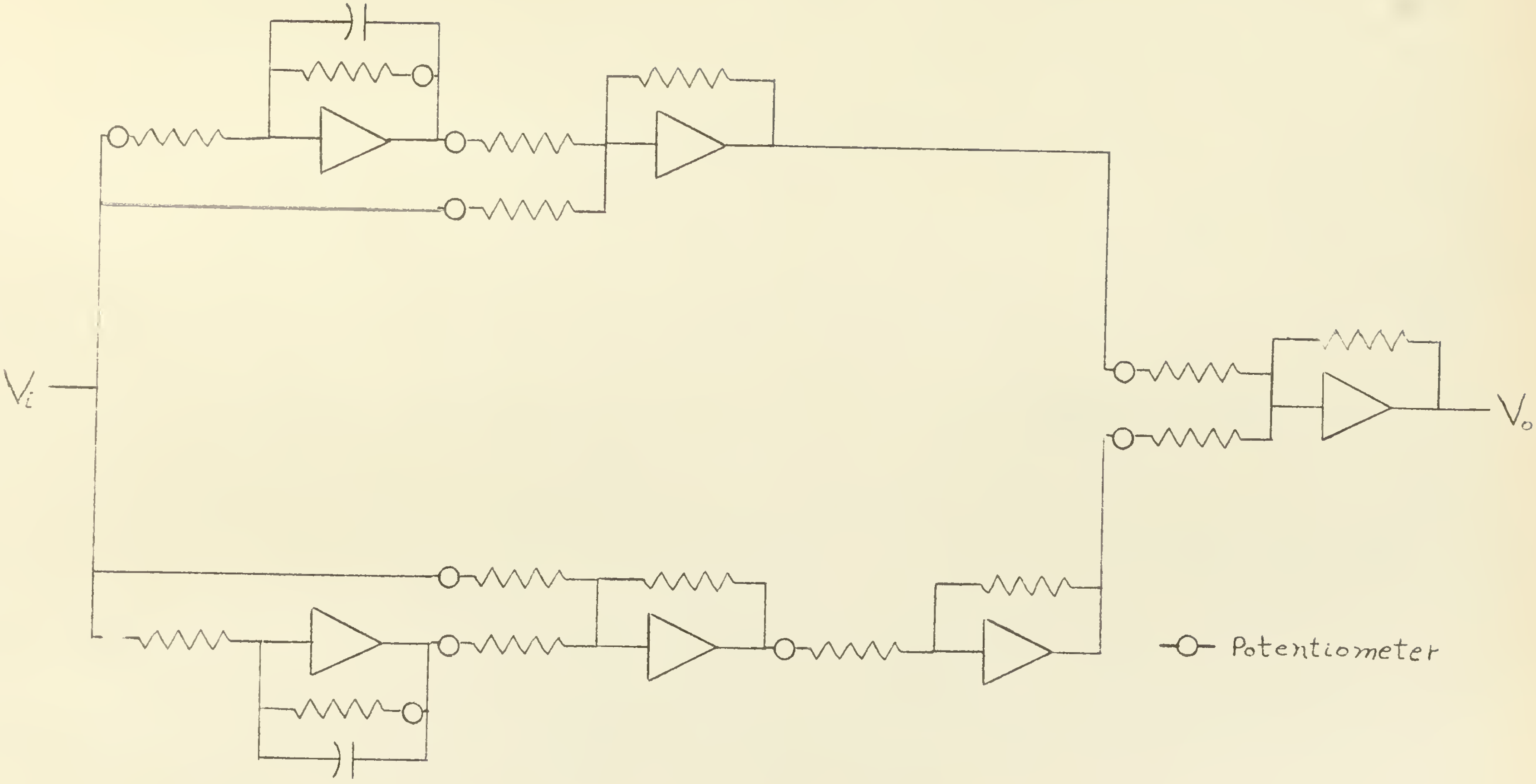


Fig. A-1. Analog computer circuit simulating the lead-lag feed forward difference compensator.

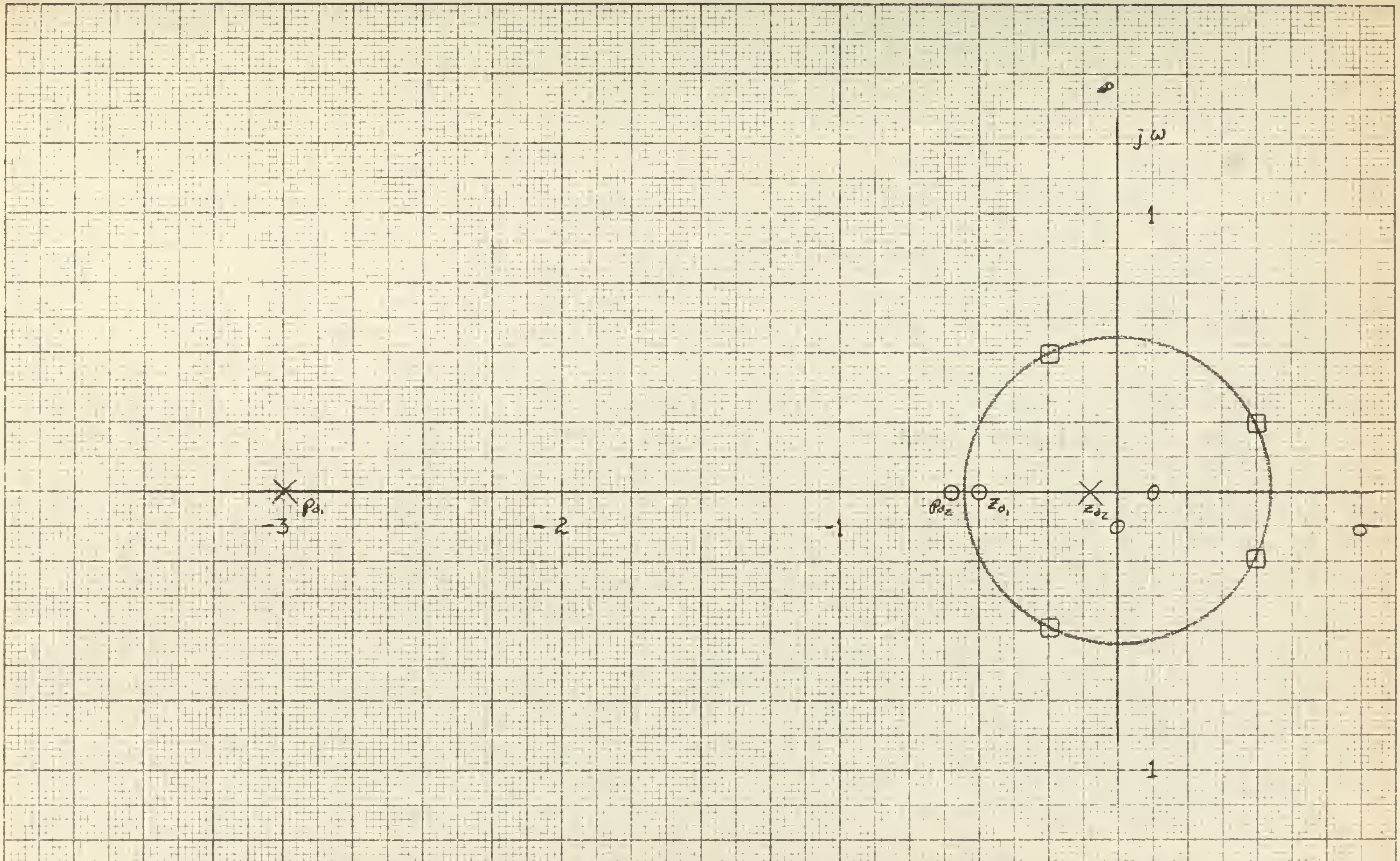


Fig. A-2. Calculated root locus circle of the simulated lead-lead feed forward difference compensator.

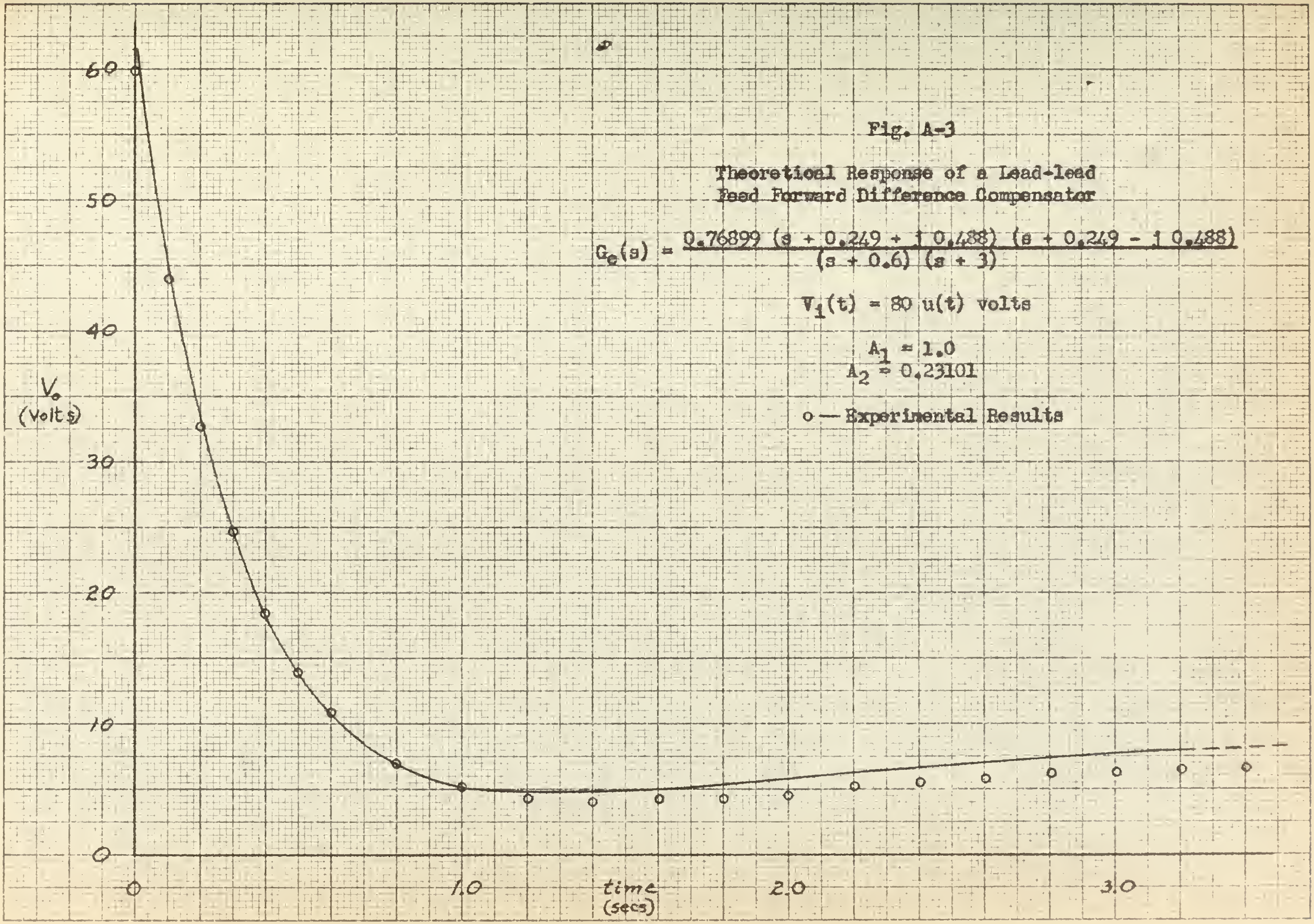


Fig. A-3

Theoretical Response of a Lead-lead
Feed Forward Difference Compensator

$$G_c(s) = \frac{0.76899\ (s + 0.249 + j\,0.488)\ (s + 0.249 - j\,0.488)}{(s + 0.6)\ (s + 3)}$$

$$V_1(t) = 80\ u(t) \text{ volts}$$

$$A_1 = 1.0$$
$$A_2 = 0.23101$$

o — Experimental Results

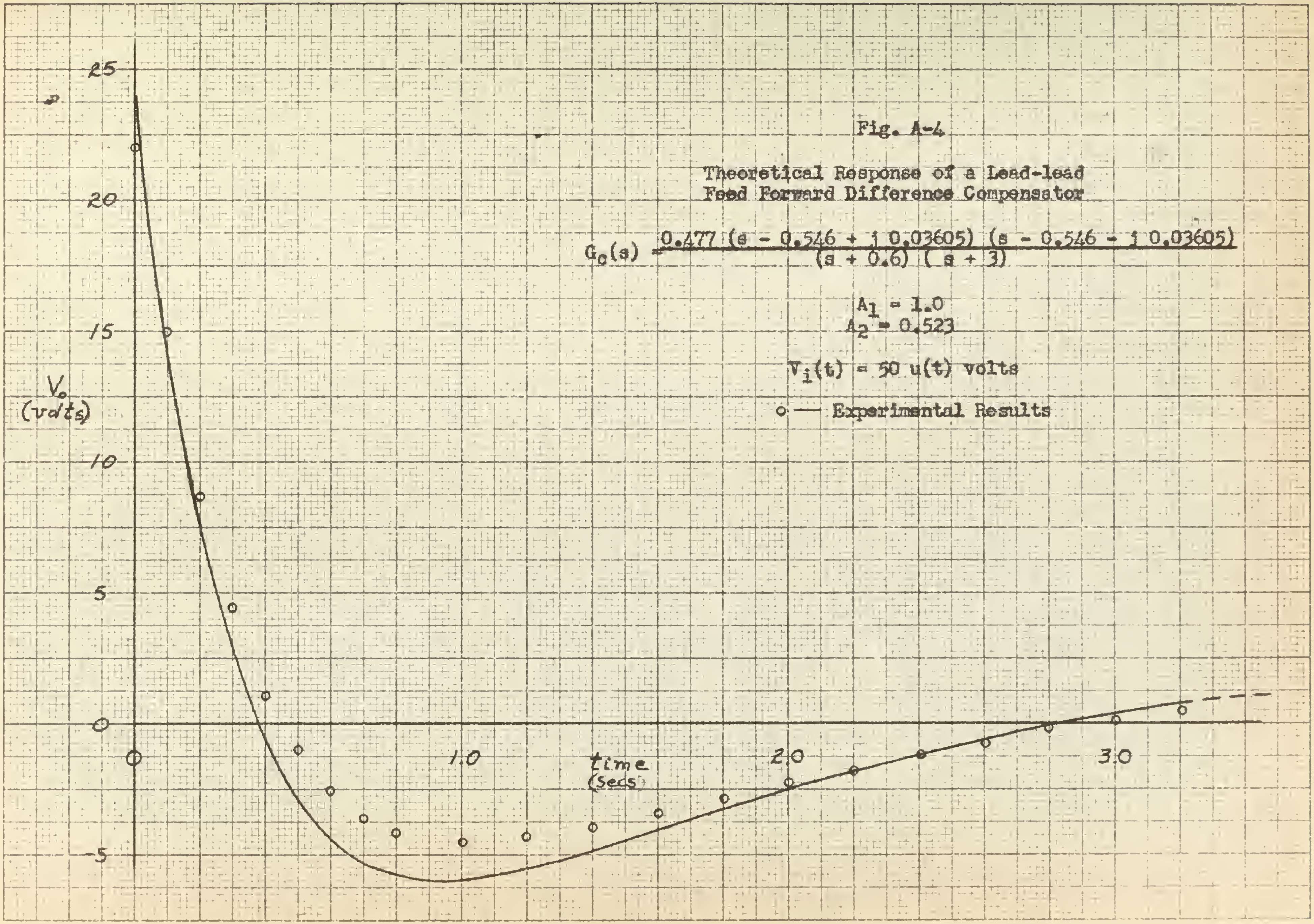


Fig. A-4

Theoretical Response of a Lead-lead Feed Forward Difference Compensator

$$G_c(s) = \frac{0.477\,(s - 0.546 + j\,0.03605)\,(s - 0.546 - j\,0.03605)}{(s + 0.6)\,(s + 3)}$$

$A_1 = 1.0$
$A_2 = 0.523$

$V_i(t) = 50\,u(t)$ volts

The circle and the numerator root locus equation are plotted on Fig. A-2. The points s = -0.249 ± j 0.488 and s = 0.546 ± j 0.3605 were arbitrarily chosen as those at which complex zeros would be located to test the transient response. By calculations from Fig. A-2, the gain ratio A_1/A_2 = 4.329 was required to locate the complex zeros at the former points and A_1/A_2 = 1.913 to place the complex zeros at the latter. The theoretical and experimental responses of the circuit to a step input with the complex zeros at s = -0.249 ± j 0.488 is Fig. A-3. The theoretical transient response was calculated by taking the inverse transform of the transfer function of the compensator. This yielded

$$V_o(t) = 61.52\left[0.167 - 0.251 e^{-0.6t} + 1.084 e^{-3t}\right] u(t)$$

The theoretical and experimental responses of the circuit to a step input with the complex zeros at s = 0.546 ± j 0.03605 is Fig. A-4. The theoretical transient response was calculated by taking the inverse transform of the transfer function of the compensator. This yielded

$$V_o(t) = 23.85\left[0.167 - 0.914 e^{-0.6t} + 1.747 e^{-3t}\right] u(t)$$

Lead-lead negative feedback compensator.

This compensator is simulated by the circuit shown in Fig. A-5. This experiment proves the results of Equation 4.13 in that given two lead networks, a circle upon which complex poles may be located is specified. The exact location of the poles is fixed by the gain A_1. Using identical lead networks

$$G_d(s) = \frac{s + 0.5}{s + 3}$$

a circle with center, c = -1.75, and radius, r = 1.25 is specified by Equation 4.13.

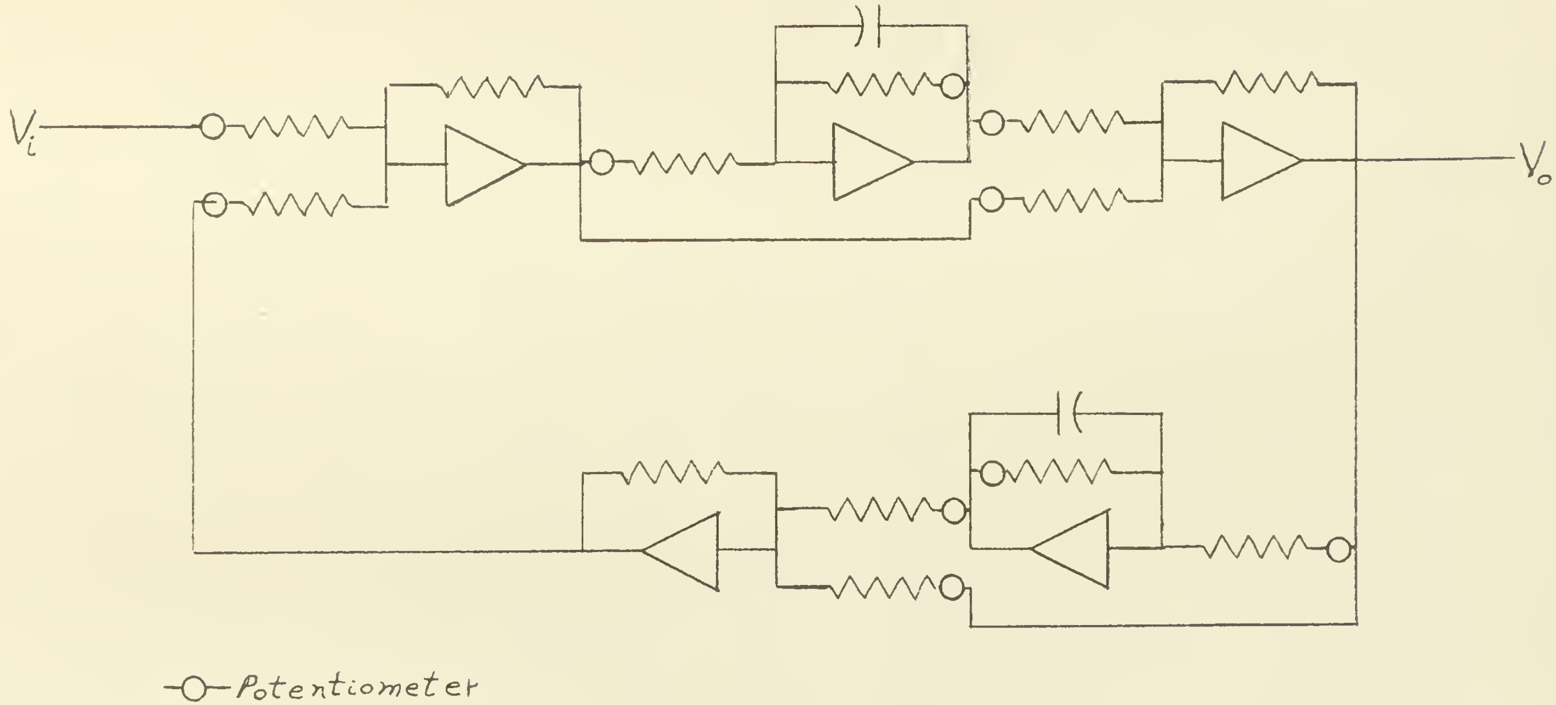


Fig. A-5. Analog computer circuit simulating the lead-lead negative feedback compensator.

The denominator root locus equation from Table 5-1 is

$$\frac{A_1(s+Z_{d_1})(s+Z_{d_2})}{(s+P_{d_1})(s+P_{d_2})} = -1$$

The circle and the denominator root locus equation are plotted on Fig. A-6. The points s = -1.75 $\pm$ j 1.25 and s = -1 $\pm$ j 1 were arbitrarily chosen as those at which complex poles would be located to test the transient response. By calculations from Fig. A-6, the gain A_1 = 1.0 was required to locate the complex poles at the former points and A_1 = 4.0 to place the complex poles at the latter. The theoretical and experimental responses of the circuit to a step input with the complex poles at s = -1.75 $\pm$ j 1.25 is Fig. A-7. The theoretical transient response was calculated by taking the inverse transform of the transfer function of the compensator. This yielded

$$V_o(t) = 25\left[0.3243 + 1.163e^{-1.75t}\sin(1.25t + 0.62)\right]u(t)$$

The theoretical and experimental responses of the circuit to a step input with the complex poles at s = -1 $\pm$ j 1 is Fig. A-8. The theoretical response was calculated by taking the inverse transform of the transfer function of the compensator. This yielded

$$V_o(t) = 16\left[0.75 + 1.77e^{-t}\sin(t + 0.14)\right]u(t)$$

Results

The remainder of the networks were not tested for their responses due to time limitations. Based on the responses obtained, it is assumed that the other networks would yield similar results since all derivations followed identical methods. It was found to be impossible to reproduce the exact theoretical transient response, however, the agreement

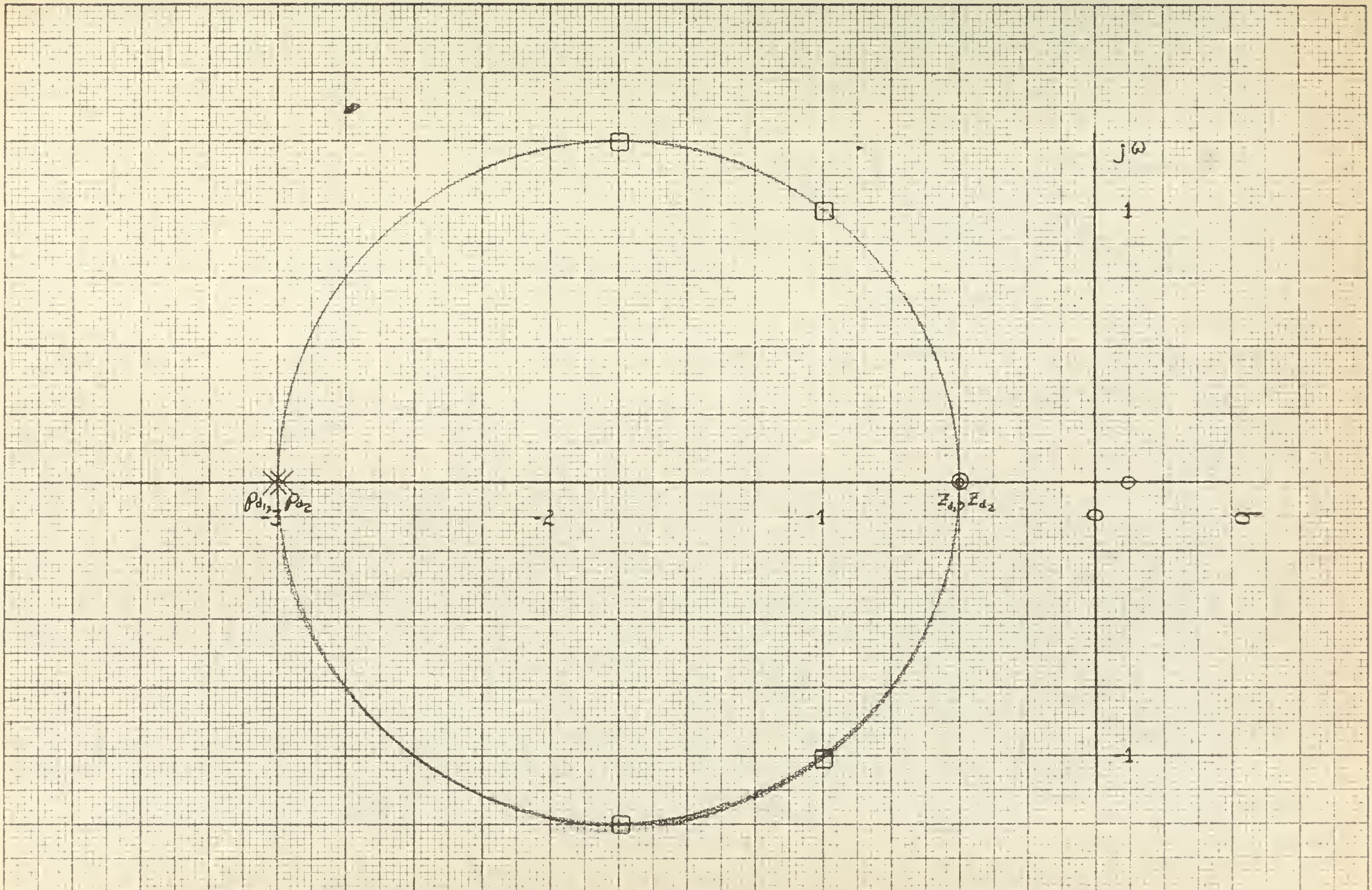


Fig. A-6. Calculated root locus circle of the simulated lead-lead negative feedback compensator.

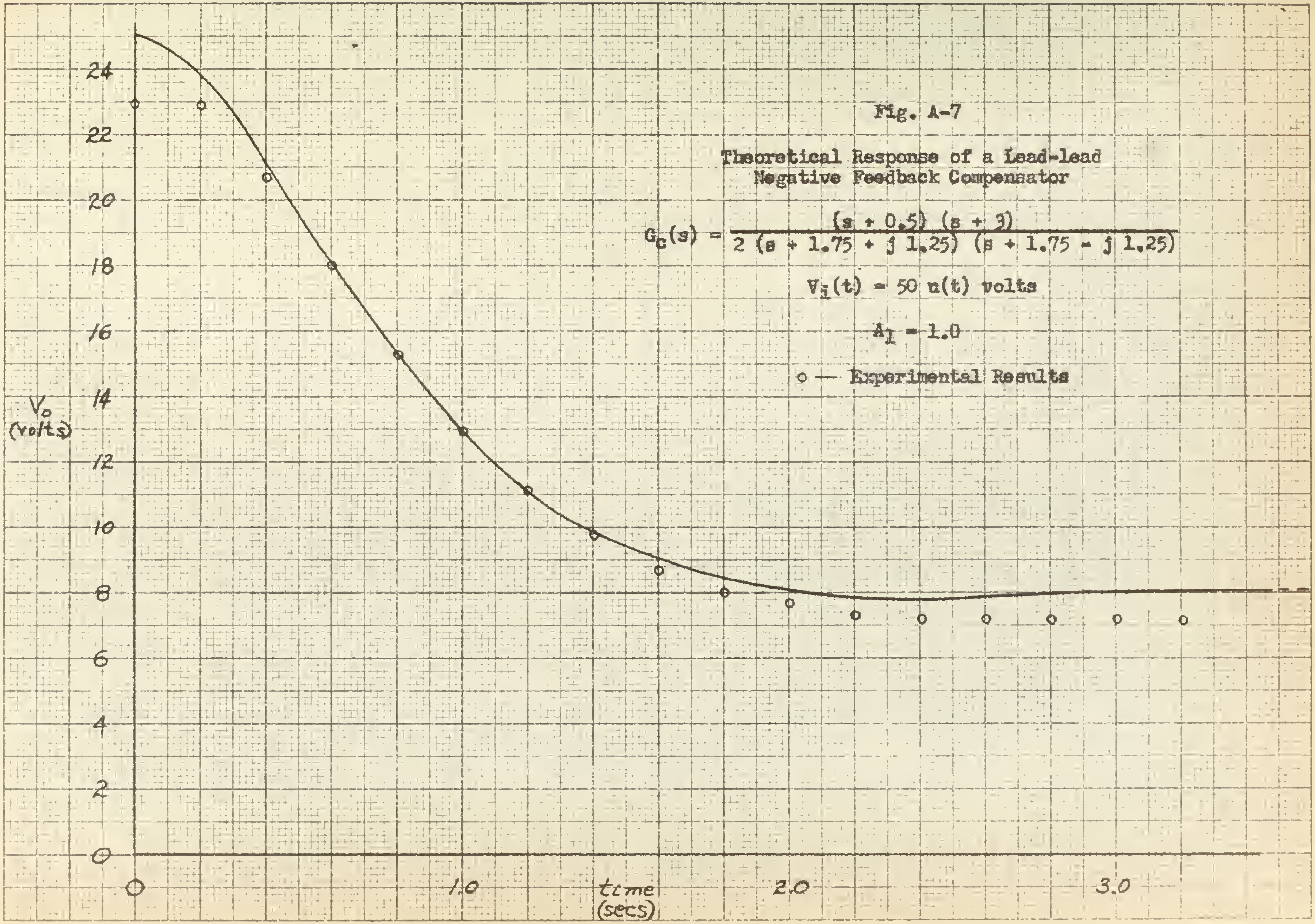


Fig. A-7

Theoretical Response of a Lead-lead Negative Feedback Compensator

$$G_c(s) = \frac{(s + 0.5)\ (s + 3)}{2\ (s + 1.75 + j\ 1.25)\ (s + 1.75 - j\ 1.25)}$$

$V_i(t) = 50\ u(t)$ volts

$A_I = 1.0$

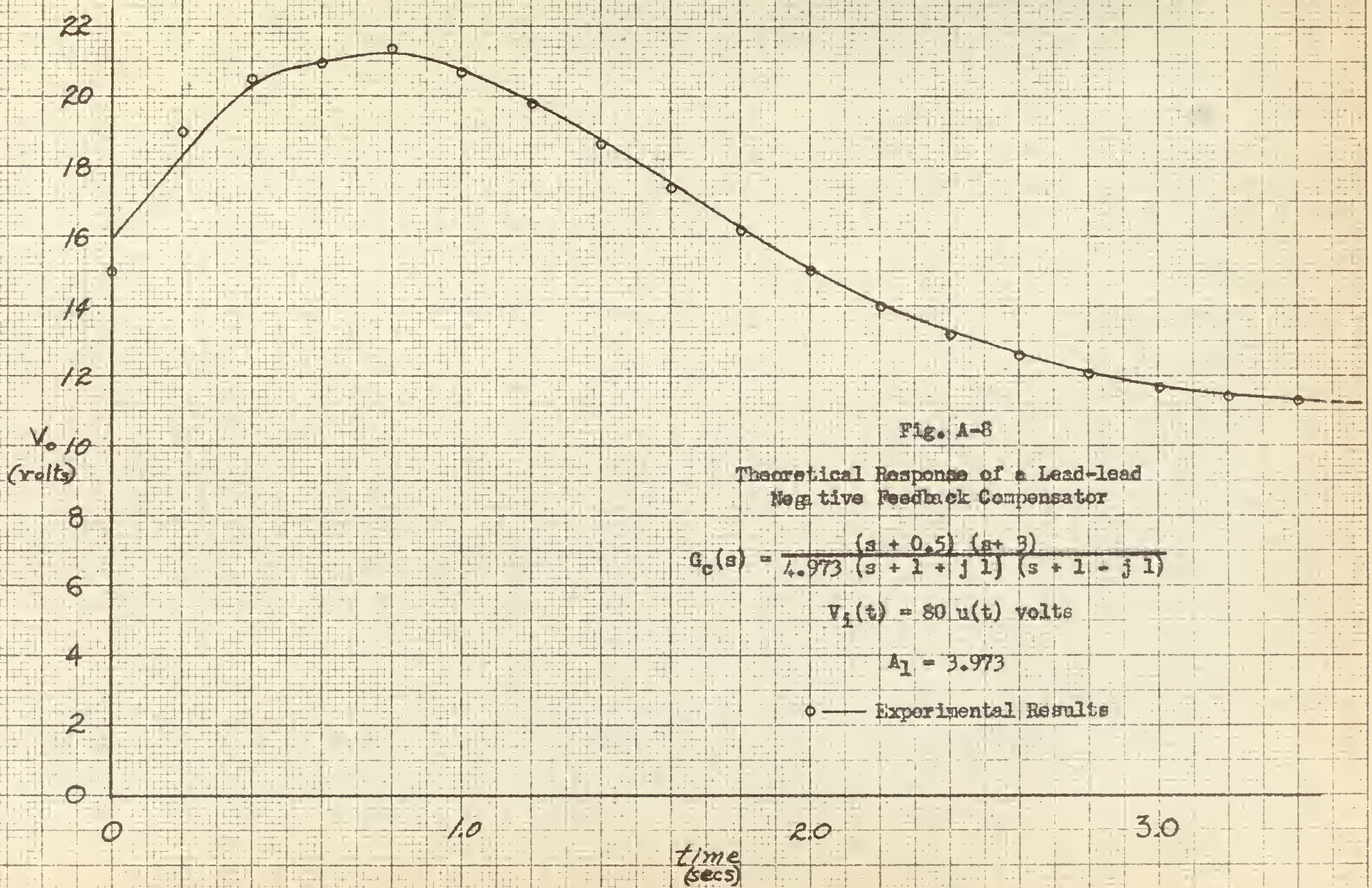


Fig. A-8

Theoretical Response of a Lead-lead
Neg tive Feedback Compensator

$$G_c(s) = \frac{(s + 0.5)\ (s + 3)}{4.973\ (s + 1 + j\ 1)\ (s + 1 - j\ 1)}$$

$V_i(t) = 80\ u(t)$ volts

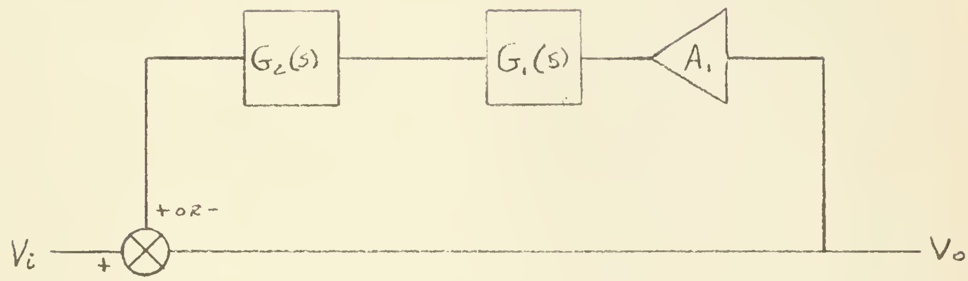
$A_1 = 3.973$

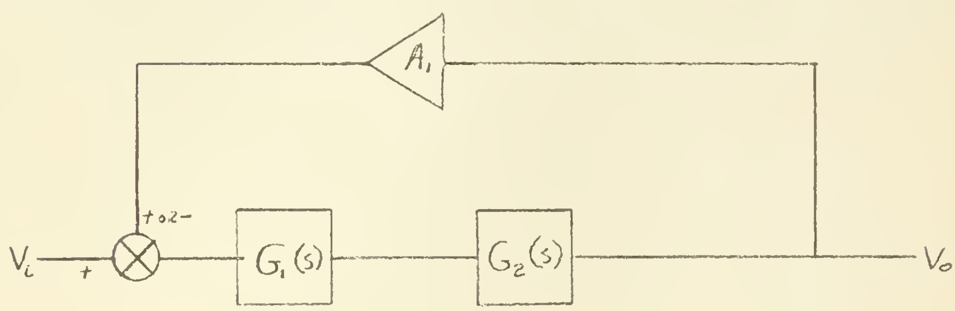
o —— Experimental Results

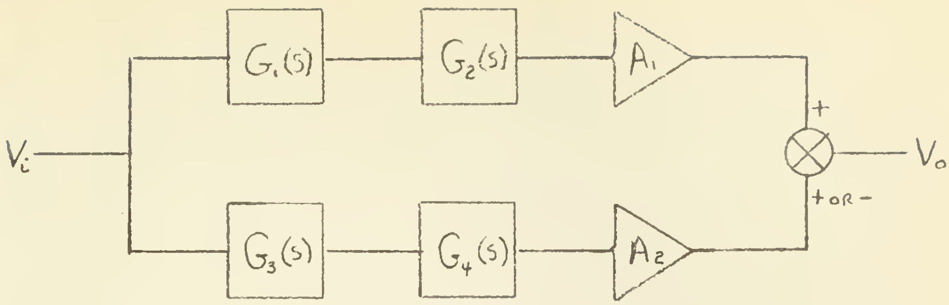
is close enough to indicate that the circuits do perform as predicted by theory.

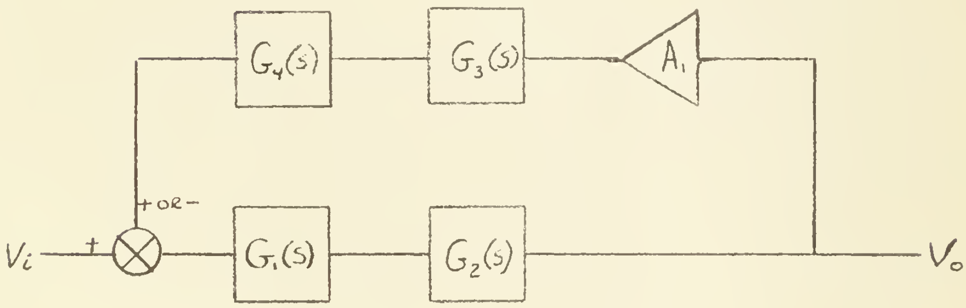
Appendix B

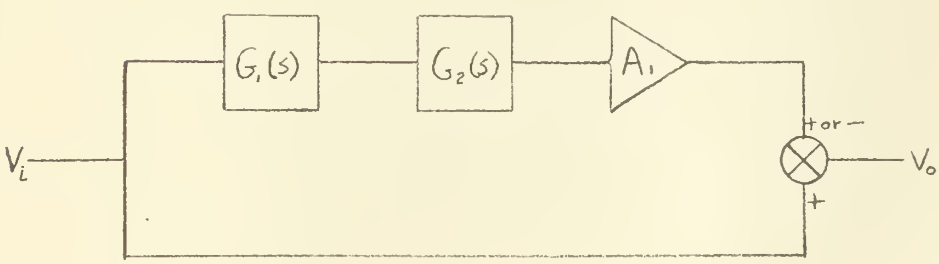
Some Networks Which May Produce Useful Results In Future Investigations of This Nature






V_i
$G_1(s)$
$G_2(s)$
A_1
+
V_o
+ OR −
$G_3(s)$
$G_4(s)$
A_2


$G_4(s)$
$G_3(s)$
A_1
+ OR −
V_i
+
$G_1(s)$
$G_2(s)$
V_o


$G_1(s)$
$G_2(s)$
A_1
V_i
+ or −
V_o
+